Final Report

ANALYSIS OF TRANSIT LINE EXTENSIONS WITH A COMBINED TRAVEL CHOICE MODEL

by

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LIST OF ABBREVIATIONS

LB	Lower Bound
BLB	Best Lower Bound
BPR	Bureau of Public Roads
CATS	Chicago Area Transportation Study
CBD	Central Business District
CTA	Chicago Transit Authority
GCU	Generalized Cost Unit
IVT	In-vehicle Travel Time
NCBD	Non-CBD
OVT	Out-of-vehicle Travel Time
PHT	Person Hours Traveled
PMT	Person Miles Traveled
TransLab	Transportation Laboratory at UIC
UIC	University of Illinois at Chicago
VHT	Vehicle Hours Traveled
VMT	Vehicle Miles Traveled

LIST OF SYMBOLS

α_i	Lagrange multiplier for origin constraint on zone i
β_j	Lagrange multiplier for destination constraint on zone j
δ_r^a	Link-route incidence (1 if link a belong to route r, 0 otherwise)
ε_b	Balancing Accuracy
ε_e	Evans Accuracy
γ_1	Cost parameter for auto IVT (GCU/minute)
γ_2	Cost parameter for auto operating cost such as the cost of gasoline (GCU/cent)
γ_3	Cost parameter for auto OVT (GCU/minute)
γ_4	Cost parameter for transit IVT (GCU/minute)
γ_5	Cost parameter for transit fare (GCU/cent)
γ_6	Cost parameter for transit OVT (GCU/minute)
γ_7	Transit bias (GCU)
λ	A variable whose value ranges from 0 to 1 and which gives a convex combination of the main and sub-problem solutions
λ_o^p	Optimal value of λ
θ_r	Lagrange multiplier for non-negativity of route flow on route r
μ	Cost sensitivity parameter (1/GCU)

$\tau^{\rho}_{i_\rho j_\rho}$	Regional network transit cost component (the same type of component as $\tau^{\sigma}_{i_\sigma j_\sigma}$) from regional network zone i_ρ to regional network zone j_ρ
$\tau^{\sigma}_{i_\sigma j_\sigma}$	Sketch network transit cost component from sketch network zone i_σ to sketch network zone j_σ
η_t	Lagrange multiplier for transit share constraint
a	Link (any link in the network)
a_i	Origin Attractiveness Factor
a_i^q	Origin attractiveness factor for zone i in balancing iteration q
$A_{i_\rho}^{i_\sigma}$	Contribution factor for regional network zone i_ρ w.r.t. sketch network zone i_σ
$A_{j_\rho}^{j_\sigma}$	Contribution factor for regional network zone j_ρ w.r.t. sketch network zone j_σ
b_j	Destination Attractiveness Factor
$\bar{b}_j$	Pre-estimated destination attractiveness factor for zone j
b_j^q	Destination attractiveness factor for zone j in balancing iteration q
$c_a(.)$	Auto IVT as an increasing function of link flow on link a (minute)
C_{ijh}	Generalized cost of travel by auto
C_{ijm}	Generalized cost of travel by mode m

C^{p}_{ijm} Main problem generalized cost of travel from zone i to zone j by mode m in Evans iteration p

C_{ijt} Generalized cost of travel by transit

c_{ijt} Transit IVT from zone i to zone j (minute)

c_t Transit Attractiveness Coefficient

F_a Free-flow travel time i.e. IVT at zero flow on link a (minute)

f_r Flow on route r (vehicles/hour)

g(.) Objective function (GCU/hour)

$\hat{g}(.)$ Partially linearized objective function, i.e. the sub-problem objective function

$g'(.)$ Gradient of g(.) at the main problem solution

G(.) Convex combination of main problem and sub-problem solutions

H Highway average vehicle occupancy (persons/vehicle)

i Trip origin zone (any zone in the network)

i_ρ Regional network trip origin zone

i_σ Sketch network trip origin zone

$I^{\rho}_{i_\sigma}$ The set of regional network zones composing the sketch network zone i_σ

j Trip destination zone (any zone in the network)

j_ρ Regional network trip destination zone

j_σ Sketch network trip destination zone

$J^{\rho}_{j_\sigma}$	The set of regional network zones composing the sketch network zone j_σ
$k_a(.)$	Auto operating cost as a function of link flow on link a (cent)
K_a	Capacity of link a (vehicles/hour)
k_{ijt}	Transit fare from zone i to zone j (cent)
L	Lagrangian of the optimization problem
m	Mode of travel (auto or transit)
N	Total number of trips (trips/hour)
p	Evans iteration number
P_i	Proportion of total person trips leaving zone i (1/hour)
$\bar{P}_i$	Observed proportion of total person trips leaving zone i (1/hour)
P_j	Proportion of total person trips entering zone j (1/hour)
$\bar{P}_j$	Observed proportion of total person trips entering zone j (1/hour)
P_{ijh}	Proportion of person trips from zone i to zone j by auto (1/hour)
P	The matrix (P_{ijh})
P_{ijm}	Proportion of person trips from zone i to zone j by mode m (1/hour)
P^{p}_{ijm}	Main problem proportion of flow from zone I to zone j by mode m in Evans iteration p
P_{ijt}	Proportion of person trips from zone i to zone j by transit (1/hour)
$\bar{P}_t$	Observed proportion of total trips made by transit over the entire network.

LIST OF SYMBOLS (continued)

q	Balancing iteration no
Q^{p}_{ijm}	Sub-problem proportion of flow from zone i to zone j by mode m in Evans iteration p
$\mathbf{Q}$	The matrix (Q^{p}_{ijm})
r	Route (any valid route in the network)
R_{ij}	Set of all the routes from zone i to zone j
T_{ij}	Number of person trips from zone i to zone j (trips/hour)
u_{ijh}	Lagrange multiplier for conservation of auto route flow from zone i to zone j
v_a	Flow on link a (vehicles/hour)
$\mathbf{v}$	The vector (v_a)
v^{p}_{a}	Main problem flow on link a in Evans iteration p
w_{ijh}	Auto OVT from zone i to zone j (minute)
w_{ijt}	Transit OVT from zone i to zone j (minute)
z^{p}_{a}	Sub-problem flow on link a in Evans iteration p
$\mathbf{z}$	The array (z^{p}_{a})

SUMMARY

This study examines the effect of alternative origin-destination constraints in the formulation of the Combined Travel Choice Model on transit ridership predictions for transit service scenarios involving extensions to new locations in a metropolitan area. Origin-destination choice is a part of the Combined Model, which can be formulated to include origin constraints, destination constraints or both. If the transit network were extended to serve outlying zones directly, the formulation of the Combined Model with regard to the application of origin constraints and/or destination constraints could affect the prediction of the effect of the extension on transit ridership. This study examines the extent and nature of such effects.

The research was conducted in several sequential steps. First, an existing C language computer code, developed at the Transportation Laboratory at the University of Illinois at Chicago (UIC), was enhanced to solve different formulations of the Combined Model. Two formulations were relevant to this study: one with both origin constraints and destination constraints, and another with only origin constraints. Then the computer implementation of the different Combined Model formulations was calibrated. The transportation network was divided into different transit corridors according to the market areas of the existing transit service, which was the basis of the transit ridership structure used in this study. Additional modules to the computer code were added to compute transit ridership in different corridors.

Two sets of transit input data were available for conducting the computational experiments. Both sets of transit data were validated in order to select a suitable one to be used in the study. Two existing transit lines in one of the corridors were hypothetically extended and the selected set of transit data was modified to reflect the effect of the extensions. The original and the extended transit networks were used to solve different formulations of the Combined Mode. These solutions included transit ridership in the different corridors, which was then extensively analyzed. Finally, the effects of the capacity of the highway network on the effect of the transit service extension were investigated.

The Model generally showed a logical response with respect to the effect of transit line extension. The prediction of the changes in transit ridership due to the extensions was different, both quantitatively and qualitatively, in the case of the two representations of origin and destination constraint used in the study. The research findings show that the degree of disagreement between the predictions of the two model grows as the accompanying highway network becomes more congested. It was also found that interpretation of the model prediction must be done carefully, especially in the case of the Combined Model with both origin and destination constraints. The study showed that the effect of transit line extensions is confined to the area served by the extensions when only origin constraints are used. When both origin and destination constraints are used, the effect inappropriately spills over the entire region leading to implausible results. Scenario analysis involving minor network changes should be conducted with caution with regard to the use of origin and destination constraints.

1. INTRODUCTION

1.1 Interaction Between Transit and Auto Networks and Transit Service Extensions

Transportation networks used by travel forecasting models are basically composed of zones, nodes and links. Zones are the origins and destinations of trips made over the network; nodes represent intersections of the links. Links represent the network facilities and services over which zone-to-zone travel takes place. In a typical model, there is no physical interaction between the auto and transit networks. In fact in order to predict origin-destination flow by transit, the representation of transit network itself can be altogether omitted.

Exclusion of transit links and nodes from the transportation network can be accomplished by providing specific transit inputs to a travel forecasting model. Transit inputs consist of all the components of cost incurred by travelers using transit between every pair of zones. These cost components are determined such that the total cost of travel by transit is a minimum for each zone pair. These transit data are commonly known as transit cost components, and consist of in-vehicle travel time (IVT), out-of-vehicle travel time (OVT) and fare. Transit cost components result from a separate analysis performed before solving the travel forecasting model. This approach simplifies the solution of travel forecasting models by obviating the assignment of transit flows to routes. Since transit travel takes place along predefined fixed routes with well-defined service characteristics and are considered to be unaffected by congestion effects, the separation of the computation of the transit cost components from such models is realistic.

Interaction between the auto and transit network is realized by the mode choice element of the model, which allocates trips between the auto and transit modes. The cost components of travel by auto are IVT, OVT, operating cost and other costs such as parking fees, access time to the auto network, toll etc. Both transit and auto cost components have heterogeneous units and are not directly comparable. In order to convert all the components into a comparable unit, weights are applied to each cost component and then the weighted components are summed giving a measure of the overall cost of travel, which is called the Generalized Cost of Travel, or simply Generalized Cost. Generalized cost is comparable across different modes and origin-destination pairs. The unit of generalized cost will be called Generalized Cost unit (GCU). The weights used to calculate the generalized cost are known as Cost Parameters. The cost parameter for an auto or transit cost component indicates how onerously that individual component is perceived by travelers.

As the generalized cost of travel over the auto network increases relative to that over the transit network, the model allocates more trips to the transit network in order to minimize cost of travel. Usually all the zones in a transportation network are connected by auto network. However, many zones may not be served by transit. Travel by transit between zones not directly served by the transit network can be undertaken as follows:

1. Drive or walk to a nearby transit station, i.e. to a zone directly served by transit.
2. Take transit to a station, i.e. a zone on transit network near the destination. Transfers from one transit service to another can be made along the way.
3. Walk to the destination.

Obviously, if no direct transit service is available, a substantial amount of OVT is associated with transit trips. In general, among all the transit cost components OVT has the largest contribution to the generalized travel cost. If transit service is extended to serve new zones directly, transit OVT, and hence the generalized cost of travel, will be significantly reduced. Since cost minimization motivates the choice of mode, a transit service extension will make transit a more attractive option for travelers over auto. Consequently the model will allocate more trips to the transit network.

1.2 Origin-Destination Choice and Transit Service Extensions

As mentioned in section 1.1, the Combined Model is essentially a mathematical optimization formulation that minimizes a function of travel cost over the entire transportation network subject to certain constraints meant to assure realistic results. The objective of the Combined Model can also be interpreted as the maximization of user benefit. Achievement of this objective is subject to conservation of traffic flow and non-negativity of trip constraints. The model also has to ensure that the trips are dispersed enough throughout the zones and modes, which it does by maximizing an artificial entity termed *entropy*.

In addition to maintaining the conservation of route flow, the model may maintain any or both of the following two forms of conservation of flow: conservation of trip origins and conservation of trip destinations. Conservation of origin flow constraints, which are known as Origin Constraints, forces the model to predict origin-destination flows by different modes in such a way that the number of trips leaving each zone equals a pre-determined value for that zone. Conservation of destination flow constraints are known as Destination Constraints, which ensure that the model's trip prediction is such that the total number of trips entering a zone is equal to a pre-determined value for that zone. The numbers of trips leaving and entering the zones are called Origins and Destinations respectively. The optimality conditions for solving the Combined Model give rise to a matrix balancing algorithm, which yields origin-destination trip matrices for each mode of travel. Application of the origin constraints in the combined model formulation is tantamount to the row balancing part of the overall matrix balancing, whereas the consideration of the destination constraints translates to its column balancing phase. Regarding the origin constraints and destination constraints, four choices can be made.

1. Application of both origin constraints and destination constraints, i.e. performing both row and column balancing of the trip matrices.
2. Applying only origin constraints, which is equivalent to balancing only the rows of the trip matrices.
3. Applying only destination constraints, thereby balancing only the columns of the trip matrices.

4. Ignoring both the sets of constraints.

The trip matrix balancing phase of the solution algorithm of the Combined Model has an interesting interaction with the transit ridership prediction of the model, especially for a region with radial transit services like the Chicago metropolitan area. If the layout of the transit services, in particular long distance services, extend radially outward from some central zones like the Central Business District (CBD), towards the peripheral zones in the network, the region can be logically divided into radial corridors. Each corridor can be regarded as the market area for a group of transit lines located close to each other. For such a corridor, the relative attractiveness of transit will increase to many travelers in the zones benefiting from the extension. Naturally, an increase in transit ridership in the corridor encompassing the extended transit line(s) is expected. However, transit ridership in the other corridors should not be affected. Each group of lines should only affect the travel choice of the travelers in its own corridor. However, depending on the application of origin constraints and destination constraints, the model might not produce results in agreement with this logical expectation.

As explained earlier, if both origin constraints and destination constraints are employed, the model will maintain the destination (column) total of the trip balanced so that it produces the pre-determined origins and destinations. As a result, an increase in transit ridership in the corridor with the transit line extension can only be accompanied by a decrease in transit ridership in some or all of the other corridors. Naturally, such a prediction from the model could be implausible and misleading. Since destination constraints are the root of this undesirable behavior of the model, an intuitive solution to this apparent anomaly is the use of the formulation of combined model without any destination constraint. However, the extent of the undesirable response of the Combined Model formulation with origin constraints and destination constraints, later referred to as the 2D Combined Model, must be examined. The differences that the combined model with only origin constraints, which will be subsequently called the 1D Combined Model, offers should also be investigated. The current research is aimed at studying these issues.

In fact, this study was initiated in response to such an experience encountered by the Regional Transportation Authority (RTA) of Northeastern Illinois. While investigating the effect of extending the suburban commuter rail services in the Chicago region, RTA noticed that although proposed commuter rail line extensions yielded increased boarding at most stations, their model predicted decreases in boarding at some stations along rail lines in other corridors. This intriguing observation motivated this extensive study of the behavior of 1D and 2D Combined Models in response to transit line extensions.

1.3 Thesis Overview

The rest of the thesis consists of four chapters. Chapter 2 presents the formulation, derivation of the optimality conditions and solution algorithm of 1D and 2D Combined Models. Chapter 3 introduces the network and the corridors used in this study and describes implementation of the models and the procurement and preparation of the transit data. The process of transit data validation and calibration of the model implementation are also discussed in Chapter 3. The implementation of transit line extension, design of computational experiments and the investigation of the interaction between the trip matrix balancing and transit line extension are discussed in Chapter 4. Chapter 4 also presents the analysis of the results and the findings of the research. Finally, conclusions derived from this study and the suggestions for future research are given in Chapter 5.

2. COMBINED TRAVEL CHOICE MODEL

2.1 Development of the Combined Model

The general problem of predicting travel and location choice in an urban transportation system includes options such as mode, route, and location that depend on travel time and cost. This kind of transportation system can be analyzed with network equilibrium models. Such a network equilibrium model was first formulated and analyzed in the 1950s. Wardrop (1952) stated the then-intuitive criteria governing the question of route choice. According to these criteria, the journey time on all the routes actually used are equal and not more than those which would be experienced by a single vehicle on any unused route. Beckmann, McGuire and Winsten (1956) formulated a variable demand network equilibrium model as a convex programming problem assuming the cost of travel on each link of the network to be an increasing function of the link flow. The optimality conditions of the model correspond to the user optimal route choice criteria of Wardrop. However, no solution algorithm was presented for this model.

Dafermos (1968) proposed two route flow based algorithms for the fixed demand network equilibrium problem. LeBlanc (1973) presented a link flow based linearization algorithm for the fixed demand problem, which was computationally more tractable than other algorithms. Murchland (1966) first pointed out that the computation of trip distribution and assignment might be combined, which came to be known as the Combined Model. Although the variable demand model of Beckmann, McGuire and Winsten was equivalent to the combined model that Murchland suggested, the correspondence was established by Evans (1973).

The development outlined above was largely confined in the academic arena. In the world of professional transportation planning, the widely practiced travel forecasting methodology followed a sequential four-step procedure: trip generation, trip distribution, mode split, and trip assignment. This approach led to the development of individual models for each of the four steps, which were linked but formulated and solved separately.

However, in real world situations travelers make decisions about where and how to travel neither separately nor sequentially. In fact, travelers take into account all the available choices and information before deciding whether to make a trip, and if a trip is to be made, where to travel and what mode and route to use. Clearly sequential models fail to capture the integral nature of travel phenomena. The assumptions, formulations, parameters and variables of different models used in the four-step travel forecasting procedure are inconsistent which undermines the stability and reliability of the forecast. By combining trip distribution, mode split, and trip assignment steps, the Combined Model seeks to produce more consistent and reliable predictions.

In a Combined Model, a common objective function representing a weighted sum of a function of link costs and negative entropy is minimized, subject to the conservation of traffic flow and non-negativity of flow yielding travel demands by different modes between all the zone pairs in the network. Evans formulated a Combined Model for trip distribution and assignment and also proposed a partial linearization algorithm for solving the model. Erlander further enhanced the model by interpreting the entropy term in Evans objective function as a constraint accounting for the dispersion of trips among different origin-destination pairs as observed in travel data.

2.2 General Characteristics of a Combined Model

A Combined Model is a constrained optimization problem in which a convex function is minimized subject to definitional and behavioral constraints. Different formulations of Combined Models exist; this chapter presents the two versions of the Combined Model used in this study, namely the 2D Combined Model and the 1D Combined Model.

The formulation of a Combined Model is based on the assumption that travelers seek routes and modes that minimize their own perceived cost of travel which is composed of different travel cost components such as IVT, OVT, monetary cost, operating cost etc. Since routes between different origin-destination pairs are composed of network links, the cost of using a route, known as *route cost*, can be computed by summing the travel cost on individual links, called *link cost*, along the route. In the formulation of the Combined Model it is assumed that the cost of a link depends only on its own flow of traffic, termed *link flow*, and that link cost is an increasing function of link flow. The later assumption reflects the effect of road congestion on travel cost.

The objective function used in the Combined Model is artificial and does not convey any direct economic or physical meaning. Competition among different routes and modes is based on the generalized cost of travel in the model. In practice the model is usually employed to analyze a one-hour period and all the trips involved are defined as hourly entities.

2.3 A Combined Model with Both Origin Constraints and Destination Constraints

The 2D Combined Model, i.e. the doubly-constraint combined model, can be mathematically formulated as follows.

$$\min g(\mathbf{v},\ \mathbf{P}) = \frac{H}{N}\gamma_1 \sum_a \int_0^{v_a} c_a(x)dx + \frac{1}{N}\gamma_2 \sum_a \int_0^{v_a} k_a(x)dx + \gamma_3 \sum_i \sum_j P_{ijh} w_{ijh} +$$
$$\gamma_4 \sum_i \sum_j P_{ijt} c_{ijt} + \gamma_5 \sum_i \sum_j P_{ijt} k_{ijt} + \gamma_6 \sum_i \sum_j P_{ijt} w_{ijt} + \gamma_7 \sum_i \sum_j P_{ijt} +$$
$$\frac{1}{\mu} \sum_i \sum_j \sum_m P_{ijm} \ln \frac{P_{ijm}}{\overline{P}_i \overline{P}_j} \qquad (2.1)$$

Subject to

$$\sum_{r \in R_{ij}} f_r = \frac{N}{H} P_{ijh} + T_{ij} : \qquad \forall i, \forall j \qquad (2.2)$$

$$\sum_j \sum_m P_{ijm} = \overline{P_i} : \qquad \forall i \qquad (2.3)$$

$$\sum_i \sum_m P_{ijm} = \overline{P_j} : \qquad \forall j \qquad (2.4)$$

$$f_r \geq 0: \qquad \forall r \qquad (2.5)$$

By definition

$$v_a = \sum_i \sum_j \sum_{r \in R_{ij}} f_r \delta_r^a: \qquad \forall a \qquad (2.6)$$

Where

a	=	link (any link in the network)
i	=	trip origin zone (any zone in the network)
j	=	trip destination zone (any zone in the network)
r	=	route (any valid route in the network)
m	=	mode of travel (auto or transit)
g(.)	=	objective function (GCU/hour)
v_a	=	flow on link a (vehicles/hour)
v	=	the vector (v_a)
P_{ijm}	=	proportion of person trips from zone i to zone j by mode m (1/hour)
P	=	the matrix (P_{ijm})
H	=	average auto occupancy (persons/vehicle)
N	=	total number of person trips (trips/hour)
γ_1	=	cost parameter for auto IVT (GCU/minute)
$c_a(.)$	=	auto IVT as an increasing function of link flow on link a (minute)
γ_2	=	cost parameter for auto operating cost such as the cost of gasoline (GCU/cent)
$k_a(.)$	=	auto operating cost as a function of link flow on link a (cent)
γ_3	=	cost parameter for auto OVT (GCU/minute)
P_{ijh}	=	proportion of person trips from zone i to zone j by auto (1/hour)

w_{ijh} = auto OVT from zone i to zone j (minute)

γ_4 = cost parameter for transit IVT (GCU/minute)

P_{ijt} = proportion of person trips from zone i to zone j by transit (1/hour)

c_{ijt} = transit IVT from zone i to zone j (minute)

γ_5 = cost parameter for transit fare (GCU/cent)

k_{ijt} = transit fare from zone i to zone j (cent)

γ_6 = cost parameter for transit OVT (GCU/minute)

w_{ijt} = transit OVT from zone i to zone j (minute)

γ_7 = transit bias (GCU)

μ = cost sensitivity parameter (1/GCU)

P_i = proportion of total person trips leaving zone i (1/hour)

P_j = proportion of total person trips entering zone j (1/hour)

R_{ij} = set of all the routes from zone i to zone j

f_r = flow on route r (vehicles/hour)

T_{ij} = number of truck trips from zone i to zone j (trips/hour)

$\overline{P}_i$ = observed proportion of total person trips leaving zone i (1/hour)

$\overline{P}_j$ = observed proportion of total person trips entering zone j (1/hour)

δ_r^a = link-route incidence for link a and route r (1 if link a belongs to route r, 0 otherwise).

Depending on the performance and convenience of transit relative to auto, travelers may have positive or negative attitude towards the use of transit. This aspect is reflected by the use of transit bias (γ_7) in the formulation. Transit bias serves as a measure of subsidy, when the bias value is negative, or cost, if bias has positive value, associated with transit travel as perceived by the travelers on average over the entire network. The bias also represents variables omitted from the formulation because of lack of data or ability to measure.

The meaning of each term in the objective function, from the leftmost to the rightmost, is explained in Table I. The last term, known as Entropy, requires additional explanation. The formulation of the Combined Model assumes that the travelers have perfect knowledge of all the costs associated with their origin-destination, mode and route choices and they use this information to minimize their travel costs. However, this is not strictly true in reality due to imperfect information, personal convenience or comfort etc. These effects result in some dispersion of trips to higher-cost origin-destinations, modes and routes. The entropy term accounts for this dispersion. Division by $(\overline{P}_i \overline{P}_j)$ in the logarithmic part of this term reflects prior knowledge about trip origins and destinations.

The constraint set in Equation (2.2) ensures that flow over all routes between each pair of zones sum to the total flow between that zone pair. These are known as Conservation of Route Flow constraints. Constraints in Equation (2.3) are the origin constraints, which force the model to balance its output trip matrices so that the number of trips leaving each zone equals some specified value for that zone. Similarly, the constraints in Equation (2.4), which are the destination constraints, ensure that the trip matrices produced by the model are such that the number of trips entering each zone is equal to some specified value. Equation (2.5) prevents predicted trips from being negative, since negative trips

TABLE I

INTERPRETATION OF INDIVIDUAL TERMS IN THE COMBINED MODEL OBJECTIVE FUNCTION

Term (numbered from left to right in Equation 2.1)	Interpretation
First Term	Total auto IVT (GCU/hour)
Second Term	Total auto operating cost (GCU/hour)
Third Term	Total auto OVT (GCU/hour)
Fourth Term	Total transit IVT (GCU/hour)
Fifth Term	Total transit fare (GCU/hour)
Sixth Term	Total transit OVT (GCU/hour)
Seventh Term	Contribution from transit bias (GCU/hour)
Eighth Term	Contribution from trip dispersion (GCU/hour)

have no practical meaning. Equation (2.6) is a definition that establishes the relationship between link and route flows.

In order to derive the optimality conditions for this network optimization problem its Lagrangian must be formed. The Lagrangian is defined as follows:

$$L=\frac{H}{N}\gamma_1\sum_a\int_0^{v_a}c_a(x)dx+\frac{1}{N}\gamma_2\sum_a\int_0^{v_a}k_a(x)dx+\gamma_3\sum_i\sum_j P_{ijh}w_{ijh}+\gamma_4\sum_i\sum_j P_{ijt}c_{ijt}+\gamma_5\sum_i\sum_j P_{ijt}k_{ijt}+$$

$$\gamma_6\sum_i\sum_j P_{ijt}w_{ijt}+\gamma_7\sum_i\sum_j P_{ijt}+\frac{1}{\mu}\sum_i\sum_j\sum_m P_{ijm}\ln\frac{P_{ijm}}{\overline{P_i}\,\overline{P_j}}+\sum_i\sum_j u_{ijh}\left(\frac{N}{H}P_{ijh}+T_{ij}-\sum_{r\in R_{ij}}f_r\right)+$$

$$\sum_i\alpha_i\left(\overline{P_i}-\sum_j\sum_m P_{ijm}\right)+\sum_j\beta_j\left(\overline{P_j}-\sum_i\sum_m P_{ijm}\right)+\sum_i\sum_j\sum_{r\in R_{ij}}\theta_r(-f_r) \qquad (2.7)$$

where

L = Lagrangian of the optimization problem given in Equation (2.1) through (2.6)

u_{ijh}, α_i, β_j, θ_r = Lagrange multipliers.

Optimality conditions can be derived by equating partial derivatives of L with respect to f_r, P_{ijh}, P_{ijt}, u_{ijh}, α_i, β_j to zero and using the following complementary slackness condition in the case of the non-negativity constraints.

$$\theta_r f_r = 0 \quad and \quad \theta_r \geq 0: \qquad \forall r \tag{2.8}$$

The partial derivatives w.r.t. u_{ijh}, α_i, β_j return the constraints in Equation (2.2)-(2.4) and these will be used later.

The partial derivative of L w.r.t. f_r gives the following equation.

$$\frac{\partial L}{\partial f_r} = \frac{H}{N}\gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{N}\gamma_2 \sum_a k_a(v_a)\delta_r^a - u_{ijh} - \theta_r = 0: \qquad r \in R_{ij} \tag{2.9}$$

If $f_r > 0$, i.e. route r is being used, by complementary slackness $\theta_r = 0$. Thus Equation (2.9) yields:

$$\gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{H}\gamma_2 \sum_a k_a(v_a)\delta_r^a = \frac{N}{H}u_{ijh} = \tilde{u}_{ijh} \tag{2.10}$$

If $f_r = 0$ i.e. route r is not being used, by complementary slackness $\theta_r \geq 0$. Then Equation (2.9) reduces to:

$$\gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{H}\gamma_2 \sum_a k_a(v_a)\delta_r^a = \frac{N}{H}u_{ijh} + \frac{N}{H}\theta_r = \hat{u}_{ijh} \qquad (2.11)$$

In Equation (2.10) $\tilde{u}_{ijh}$ is the flow dependent and route independent generalized cost of travel by auto on all the used routes from zone i to zone j. $\hat{u}_{ijh}$ in Equation (2.11) is the equivalent of $\tilde{u}_{ijh}$ on all unused routes. Since $\theta_r \geq 0$, clearly $\hat{u}_{ijh} \geq \tilde{u}_{ijh}$. This demonstrates that the Combined Model satisfies Wardrop's equilibrium conditions.

The partial derivative of L w.r.t. P_{ijh} gives the following.

$$\frac{\partial L}{\partial P_{ijh}} = \gamma_3 w_{ijh} + \frac{1}{\mu}\left(\ln\frac{P_{ijh}}{\overline{P}_i \overline{P}_j} + 1\right) + \frac{N}{H}u_{ijh} - \alpha_i - \beta_j = 0$$

Substituting Equation (2.10) into the above result yields

$$\Rightarrow \qquad \gamma_3 w_{ijh} + \frac{1}{\mu}\left(\ln\frac{P_{ijh}}{\overline{P}_i \overline{P}_j} + 1\right) + \gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{H}\gamma_2 \sum_a k_a(v_a)\delta_r^a - \alpha_i - \beta_j = 0 \quad (2.12)$$

Therefore, the generalized cost of travel by auto from zone i to zone j can be defined as follows.

$$C_{ijh} = \gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{H}\gamma_2 \sum_a k_a(v_a)\delta_r^a + \gamma_3 w_{ijh} \qquad (2.13)$$

Equation (2.12) and (2.13) gives the following expression.

$$C_{ijh} - \alpha_i - \beta_j + \frac{1}{\mu}\left(\ln\frac{P_{ijh}}{\overline{P}_i \overline{P}_j} + 1\right) = 0$$

$$\Rightarrow \quad P_{ijh} = \overline{P}_i \overline{P}_j \exp(-\mu C_{ijh})\exp(\mu\alpha_i - 1)\exp(\mu\beta_j)$$

$$\Rightarrow \quad P_{ijh} = a_i \overline{P}_i b_j \overline{P}_j \exp\left(-\mu C_{ijh}\right) \qquad (2.14)$$

Where,

a_i = $\exp(\mu\alpha_i - 1)$, a measure of attractiveness of zone i as an origin, or Origin Attractiveness Factor

b_j = $\exp(\mu\beta_j)$, a measure of attractiveness of zone j as a destination, or Destination Attractiveness Factor.

Again, the partial derivative of L w.r.t. P_{ijt} yields:

$$\frac{\partial L}{\partial P_{ijt}} = \gamma_4 c_{ijt} + \gamma_5 k_{ijt} + \gamma_6 w_{ijt} + \gamma_7 + \frac{1}{\mu}\left(\ln\frac{P_{ijh}}{\overline{P}_i \overline{P}_j} + 1\right) - \alpha_i - \beta_j = 0$$

$$\Rightarrow \quad P_{ijt} = \overline{P}_i \overline{P}_j \exp\left(-\mu C_{ijt}\right)\exp\left(\mu\alpha_i - 1\right)\exp\left(\mu\beta_j\right)$$

$$\Rightarrow \quad P_{ijt} = a_i \overline{P}_i b_j \overline{P}_j \exp\left(-\mu C_{ijt}\right) \qquad (2.15)$$

Where

$$C_{ijt} = \gamma_4 c_{ijt} + \gamma_5 k_{ijt} + \gamma_6 w_{ijt} + \gamma_7 \qquad (2.16)$$

= generalized cost of travel by transit from zone i to zone j.

Equation (2.3), which is the result of differentiating L w.r.t. α_i, can be written as follows:

$$\sum_j P_{ijh} + \sum_j P_{ijt} = \overline{P}_i$$

$$\Rightarrow \quad a_i \overline{P}_i \left(\sum_j b_j \overline{P}_j \exp\left(-\mu C_{ijh}\right)\right) + a_i \overline{P}_i \left(\sum_j b_j \overline{P}_j \exp\left(-\mu C_{ijt}\right)\right) = \overline{P}_i$$

$$\Rightarrow \quad a_i = \frac{1}{\sum_j \sum_m b_j \overline{P}_j \exp\left(-\mu C_{ijm}\right)} \qquad (2.17)$$

Partial derivation of L w.r.t. βj results in Equation (2.4) which can be rearranged as follows:

$$\sum_i P_{ijh} + \sum_i P_{ijt} = \overline{P}_j$$

$$\Rightarrow \quad b_j \overline{P}_j \left(\sum_i a_i \overline{P}_i \exp\left(-\mu C_{ijh}\right) \right) + b_j \overline{P}_j \left(\sum_i a_i \overline{P}_i \exp\left(-\mu C_{ijt}\right) \right) = \overline{P}_j$$

$$\Rightarrow \quad b_j = \frac{1}{\sum_i \sum_m a_i \overline{P}_i \exp\left(-\mu C_{ijm}\right)} \qquad (2.18)$$

Equations (2.10), (2.15), (2.16), (2.17) and (2.18) are the optimality conditions for the 2D Combined Model.

2.4 A Combined Model with Only Origin Constraints

The formulation of the Combined Model with only origin constraints, called the 1D Combined Model, is very similar to that of the above 2D Combined Model. The main difference is that the set of destination constraints Equation (2.4) is not applied and a different distribution is assumed in case of the entropy term, which uses only $\overline{P}_j$ and a destination attractiveness factor b_j. However, for completeness, the 1D Combined Model formulation and the derivation of the optimality conditions are presented in this section, which should be more or less self explanatory. The model can be formulated as follows.

$$\min g(\mathbf{v},\ \mathbf{P}) = \frac{H}{N}\gamma_1 \sum_a \int_0^{v_a} c_a(x)dx + \frac{1}{N}\gamma_2 \sum_a \int_0^{v_a} k_a(x)dx + \gamma_3 \sum_i \sum_j P_{ijh} w_{ijh} +$$

$$\gamma_4 \sum_i \sum_j P_{ijt} c_{ijt} + \gamma_5 \sum_i \sum_j P_{ijt} k_{ijt} + \gamma_6 \sum_i \sum_j P_{ijt} w_{ijt} + \gamma_7 \sum_i \sum_j P_{ijt} +$$

$$\frac{1}{\mu}\sum_i\sum_j\sum_m P_{ijm}\ln\frac{P_{ijm}}{\bar{b}_j\bar{P}_j} \qquad (2.19)$$

Subject to

$$\sum_{r\in R_{ij}} f_r = \frac{N}{H}P_{ijh} + T_{ij} : \qquad \forall i, \forall j \qquad (2.20)$$

$$\sum_j\sum_m P_{ijm} = \overline{P_i} : \qquad \forall i \qquad (2.21)$$

$$f_r \geq 0 : \qquad \forall r \qquad (2.22)$$

By definition

$$v_a = \sum_i\sum_j\sum_{r\in R_{ij}} f_r\delta_r^a : \qquad \forall a \qquad (2.23)$$

Where

$\bar{b}_j$ = pre-determined destination attractiveness factor for zone j.

The Lagrangian of the problem is given below.

$$L = \frac{H}{N}\gamma_1\sum_a\int_0^{v_a} c_a(x)dx + \frac{1}{N}\gamma_2\sum_a\int_0^{v_a} k_a(x)dx + \gamma_3\sum_i\sum_j P_{ijh}w_{ijh} + \gamma_4\sum_i\sum_j P_{ijt}c_{ijt} + \gamma_5\sum_i\sum_j P_{ijt}k_{ijt}$$
$$+\gamma_6\sum_i\sum_j P_{ijt}w_{ijt} + \gamma_7\sum_i\sum_j P_{ijt} + \frac{1}{\mu}\sum_i\sum_j\sum_m P_{ijm}\ln\frac{P_{ijm}}{\bar{b}j\bar{P}_j} + \sum_i\sum_j u_{ijh}\left(\frac{N}{H}P_{ijh} + T_{ij} - \sum_{r\in R_{ij}} f_r\right)$$
$$+\sum_i \alpha_i\left(\overline{P_i} - \sum_j\sum_m P_{ijm}\right) + \sum_i\sum_j\sum_{r\in R_{ij}}\theta_r(-f_r) \qquad (2.24)$$

Partial derivatives of L w.r.t. route flow yield:

$$\frac{\partial L}{\partial f_r} = \frac{H}{N}\gamma_1\sum_a c_a(v_a)\delta_r^a + \frac{1}{N}\gamma_2\sum_a k_a(v_a)\delta_r^a - u_{ijh} - \theta_r = 0 : \qquad r\in R_{ij} \qquad (2.25)$$

When route r is being used, by complementary slackness $\theta_r = 0$. Hence Equation (2.9) becomes:

$$\gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{H}\gamma_2 \sum_a k_a(v_a)\delta_r^a = \frac{N}{H}u_{ijh} = \tilde{u}_{ijh} \qquad (2.26)$$

If route r is not being used, by complementary slackness $\theta_r \geq 0$. Consequently Equation (2.9) gives:

$$\gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{H}\gamma_2 \sum_a k_a(v_a)\delta_r^a = \frac{N}{H}u_{ijh} + \frac{N}{H}\theta_r = \hat{u}_{ijh} \qquad (2.27)$$

Taking partial derivative of L w.r.t. P_{ijh}:

$$\frac{\partial L}{\partial P_{ijh}} = \gamma_3 w_{ijh} + \frac{1}{\mu}\left(\ln\frac{P_{ijh}}{\bar{b}_j \bar{P}_j} + 1\right) + \frac{N}{H}u_{ijh} - \alpha_i - \beta_j = 0$$

$$\Rightarrow \qquad \gamma_3 w_{ijh} + \frac{1}{\mu}\left(\ln\frac{P_{ijh}}{\bar{b}_j \bar{P}_j} + 1\right) + \gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{H}\gamma_2 \sum_a k_a(v_a)\delta_r^a - \alpha_i - \beta_j = 0$$

By Equation (2.26)

$$\Rightarrow \qquad C_{ijh} - \alpha_i - \beta_j + \frac{1}{\mu}\left(\ln\frac{P_{ijh}}{\bar{b}_j \bar{P}_j} + 1\right) = 0$$

$$\Rightarrow \qquad P_{ijh} = \bar{b}_j \bar{P}_j \exp(-\mu C_{ijh})\exp(\mu\alpha_i - 1)\exp(\mu\beta_j)$$

$$\Rightarrow \qquad P_{ijh} = a_i \bar{P}_i \bar{b}_j \bar{P}_j \exp(-\mu C_{ijh}) \qquad (2.28)$$

Taking partial derivative of L w.r.t. P_{ijt}:

$$\frac{\partial L}{\partial P_{ijt}} = \gamma_4 c_{ijt} + \gamma_5 k_{ijt} + \gamma_6 w_{ijt} + \gamma_7 + \frac{1}{\mu}\left(\ln\frac{P_{ijh}}{\bar{b}_j \bar{P}_j} + 1\right) - \alpha_i - \beta_j = 0$$

$$\Rightarrow \quad P_{ijt} = \bar{b}j\bar{P}_j \exp(-\mu C_{ijt})\exp(\mu\alpha_i - 1)\exp(\mu\beta_j)$$

$$\Rightarrow \quad P_{ijt} = a_i \bar{P}_i \bar{b}_j \bar{P}_j \exp(-\mu C_{ijt}) \qquad (2.29)$$

Equation (2.3), which is equivalent to the derivation of L w.r.t. α_i, can be reorganized as follows:

$$\sum_j P_{ijh} + \sum_j P_{ijt} = \bar{P}_i$$

$$\Rightarrow \quad a_i \bar{P}_i\left(\sum_j \bar{b}_j \bar{P}_j \exp(-\mu C_{ijh})\right) + a_i \bar{P}_i\left(\sum_j \bar{b}_j \bar{P}_j \exp(-\mu C_{ijt})\right) = \bar{P}_i$$

$$\Rightarrow \quad a_i = \frac{1}{\sum_j \sum_m \bar{b}_j \bar{P}_j \exp(-\mu C_{ijm})} \qquad (2.30)$$

Equations (2.26), (2.28), (2.29) and (2.30) are the optimality condition for the 1D Combined Model.

2.5 General Overview of the Solution Algorithm for the Combined Model

The Combined model is solved using a generalization of the algorithm proposed by Evans (1976). This algorithm is iterative and based on partial linearization of the objective function. The objective function is said to be partially linearized in the sense that although it is a function of both the link flows and the origin-destination flows, linearization is performed w.r.t. the link flows only.

The Evans algorithm starts at an arbitrary point, known as the Initial Solution, in the solution space bounded by the constraints and uses this partially linearized objective function to find a feasible descent direction. A point in the solution space is a feasible solution to the optimization problem that constitutes the Combined Model and is essentially a set of link flows and origin-destination flows. A solution is said to be feasible if it satisfies all the constraints of the problem. Thus, the solution space is the collection of all the feasible solutions to the network optimization problem. Clearly, the objective function value at a solution point is the quantity computed by using the link and origin-destination flow values from the solution to the objective function. The optimal solution is the feasible solution where the objective function has the lowest possible value under the constraints set in the problem formulation. A feasible descent direction is the one that leads to a new point in the solution space, from the current feasible solution, towards which the objective function value decreases.

After finding the descent direction at the current feasible solution, the algorithm searches for a point in the solution space along the descent direction where the objective function value will be the lowest in that direction. Finding this point marks the completion of one solution iteration. This new point becomes the current feasible solution, also referred to as current solution, for the next iteration and once again the algorithm starts searching for a descent direction at this solution point. The value of the objective function at the current feasible solution during an iteration is referred to as the objective function value of that iteration.

As the solution iterations, known as Evans Iterations, proceed, the solution to the Combined Model converges towards the optimal point. If the solution reaches the optimal point, no descent direction can be found and the difference between the objective function value in the previous Evans iteration and that in the current Evans iteration will be zero. However, due to the limitations of the computational accuracy of computers, such a point may never be reached; for practical purposes such an exact convergence is not necessary. Consequently the Evans algorithm stops when the relative difference or gap between the objective function value and a lower bound defined below reaches a predefined limit.

To be more concrete, the descent direction is found by linearizing the objective function w.r.t. the link flows at the current feasible solution and minimizing this partially linearized function subject to the original set of constraints. In the parlance of the Evans algorithm, minimization of the original objective function is known as the Main Problem and that of the partially linearized objective function is called the Subproblem. Since both the main problem and the subproblem are subject to the same set of constraints, any feasible solution to the subproblem is also a feasible solution to the main problem, and the optimal solution to the subproblem is a feasible solution to the main problem.

The Main Problem objective function is convex and hence the partially linearized approximation of this objective function, which is the subproblem objective function, always lies tangentially beneath it; the only point of contact between the two is the current solution to the main problem. Consequently, the optimal subproblem solution must be at or below the current Main Problem solution. If the current Main Problem solution is not the optimal solution, the optimal subproblem solution will be below the current Main Problem solution and the Main Problem objective function will decrease along the line between these two points due to its convexity. The direction from the current Main Problem solution towards the optimal subproblem solution is the so called descent direction at the current Main Problem solution.

2.6 Evans Algorithm

Evans algorithm can be best described in a step-by-step manner. Some of the steps contain sub-algorithms, which in turn can be described in terms of different steps. The rest of this section discusses the various steps involved in the Evans algorithm.

Step 0: Initialization

Based on the zero-flow link costs find the minimum-cost route between each origin-destination pair.

Calculate the origin-destination flow proportions (P_{ijm}) based on these costs, and perform an all-or-nothing assignment to the road network to obtain link flows (v_a). This is the initial solution that starts the Evans iterations.

Subsequently, the Evans iteration number is denoted by p. C_{ijm}, P_{ijm} and v_a at iteration p are denoted as C^p_{ijm}, P^p_{ijm} and v^p_a respectively. For the initial solution p is equal to zero.

Step 1: Update Link Costs

Compute link costs based on link flows v_a^p.

Step 2: Update Minimum Cost Routes

Find minimum-cost routes between all the zone pairs using the link costs calculated in Step 1.

Step 3: Find a Feasible Descent Direction

This is done in several steps as described below.

Step 3.1: Compute the generalized travel costs (C_{ijm}^p) based on the routes found in Step 2.

Step 3.2: Compute origin-destination flows corresponding to the subproblem using Equations (2.14) and (2.15) in the case the 2D Combined Model or using Equations (2.28) and (2.29) for 1D Combined Model.

Since these origin-destination flows are part of the subproblem solution, they are denoted by Q_{ijm}^p in order to distinguish them from the main problem origin-destination flows P_{ijm}^p.

Note here that (a_i), for both 1D and 2D Combined Models, and (b_j), for the 2D Combined Model, must be solved in order to calculate (Q_{ijm}^p). In the case of the 1D Combined Model (a_i) can be explicitly solved using Equation (2.30). However, for the 2D Combined Model, (a_i) and (b_j) must be solved using Equations (2.17) and (2.18).

Equations (2.17) and (2.18) imply that an iterative scheme is needed to solve (a_i) and (b_j), which is essentially as matrix balancing procedure enforcing the constraints of Equations (2.3) and (2.4). The iterations of this trip matrix balancing procedure are known as Balancing Iterations. In the subsequent discussion q will denote the number of balancing iterations and a_i and b_j in balancing iteration q is denoted as (a_i^q) and (b_i^q) respectively. The algorithm is described below.

Step 3.2.1: Initialize (a_i) to any convenient value such as 1 and calculate (b_j) using Equation (2.18) and these (a_i). These are the initial values of (a_i^q) and (b_j^q); for this iteration q = 0.

Step 3.2.2: Calculate (a_i^{q+1}) using Equation (2.17) and (b_j^q).

Step 3.2.3: Calculate (b_j^{q+1}) using Equation (2.18) and (a_i^{q+1}).

Step 3.2.4: Check whether (a_i) and (b_j) have converged. This can be done checking the following criteria:

$$\left|\frac{a_i^{q+1} - a_i^q}{a_i^q}\right| < \varepsilon_b : \qquad \forall i \qquad (2.31)$$

$$\left|\frac{b_j^{q+1} - b_j^q}{b_j^q}\right| < \varepsilon_b : \qquad \forall j \qquad (2.32)$$

where

ε_b = predefined Balancing Accuracy.

If the convergence criterion is satisfied, stop. Otherwise increment q and go to Step 2.1. However, balancing may be terminated after a certain number of balancing iteration has been performed. This provision helps to avoid an indefinite loop in the computer implementation of the balancing algorithm.

Step 3.3: Compute sub-problem link flows by assigning (Q_{ijm}^p) to the minimum-cost routes found in Step 2. Since these link flows pertain to the sub-problem solution, in order to distinguish these from the main problem solution link flows i.e. (v_a^p), these will be denoted by (z_a^p).

Step 4: This is the convergence check step. The direction from (P^p_{ijm}) and (v^p_a) towards (Q^p_{ijm}) and (z^p_a) is the descent direction. Minimize the objective function (*g(.)*) along the descent direction. Since *g(.)* is a convex function of P_{ijm} (i.e. Q_{ijm}) and v_a (i.e. z_a), this can be done by solving the following optimization problem.

$$\min_{0 \le \lambda \le 1} G(\lambda) = g(\lambda \mathbf{z} + (1-\lambda)\mathbf{v};\ \lambda \mathbf{Q} + (1-\lambda)\mathbf{P}) \qquad (2.33)$$

where

λ = a variable whose value ranges from 0 to 1 and which gives a convex combination of the main and sub-problem solutions.

$\mathbf{z}$ = the array (z^p_a)

$\mathbf{Q}$ = the matrix (Q^p_{ijm})

Equation (2.33) can be solved by a iterative algorithm that searches for the optimal value of λ in order to minimize *G(.)*. There many such algorithms and the class of this type of algorithms are generally known as Line Search Algorithm.

The value of λ that solves Equation (2.33) is known as the Step Length in the Evans algorithm and is designated by λ^p_o.

Step 5: Update P_{ijm} and v_a as follows.

$$P_{ijm}^{p+1} = \lambda_o^p Q_{ijm}^p + P_{ijm}^p \left(1 - \lambda_o^p\right): \qquad \forall i, \forall j, \forall m \qquad (2.34)$$

$$v_a^{p+1} = \lambda_o^p z_a^p + v_a^p \left(1 - \lambda_o^p\right): \qquad \forall a \qquad (2.35)$$

Check whether the convergence criterion is satisfied. The convergence is described in the next section. If the convergence conditions are satisfied, stop. Otherwise increment p and go to step 1. As with balancing iteration, the Evans algorithm might be terminated after a certain number of iterations in order to avoid an indefinite loop in the computer implementation of Evans algorithm.

2.7 Convergence Criterion for the Evans Algorithm

As noted above, the Evans algorithm makes use of a partially linearized objective function. The partially linearized objective function at the optimal sub-problem solution can be written as follows.

$$\hat{g}(\mathbf{z},\ \mathbf{Q}) = g(\mathbf{v},\ \mathbf{P}) + \nabla g(\mathbf{v},\ \mathbf{P})(\mathbf{z} - \mathbf{v}) \qquad (2.36)$$

Where,

$\hat{g}(.)$ = partially linearized objective function i.e. the sub-problem objective function at Evans iteration p at the optimal sub-problem solution, i.e. (z_a^p) and (Q_{ijm}^p).

$\nabla g(.)$ = gradient of g(.) at the main problem solution at Evans iteration p, w.r.t. link flows i.e. (v_a^p).

Because of the convexity of *g(.)*, $\hat{g}(\mathbf{z},\ \mathbf{Q})$ is always at or below $g(\mathbf{v},\ \mathbf{P})$. At the optimal solution (z_a^p) and (Q_{ijm}^p) will coincide with (v_a^p) and (P_{ijm}^p) respectively. From Equation (2.36) $\hat{g}(\mathbf{z},\ \mathbf{Q})$ is equal to $g(\mathbf{v},\ \mathbf{P})$. $\hat{g}(\mathbf{z},\ \mathbf{Q})$ is called the Lower Bound (LB) to the objective function *g(.)*. The gap or difference between $g(\mathbf{v},\ \mathbf{P})$ and $\hat{g}(\mathbf{z},\ \mathbf{Q})$ serves as a good quantity to monitor the convergence of the Evans algorithm. As the Evans iterations proceed the values of $g(\mathbf{v},\ \mathbf{P})$ decrease, whereas the values of $\hat{g}(\mathbf{z},\ \mathbf{Q})$ generally increase. However, although the decrease of the value of $g(\mathbf{v},\ \mathbf{P})$ is monotonous, the increase in the value of $\hat{g}(\mathbf{z},\ \mathbf{Q})$ is not necessarily so (Figure 1 and Figure 2). This is because although the solution space is multi-dimensional (the number of dimensions being equal to the number of links plus the product of the number of modes and the square of the number of zones) the line search in Step 4 is unidimensional. The undulation of the value of $\hat{g}(\mathbf{z},\ \mathbf{Q})$ might give rise to some oscillating behavior of the convergence monitoring criterion of the Evans algorithm. In order to obtain a more consistent convergence criterion for the algorithm, Best Lower Bound (BLB) is used instead of the current lower bound.

The best lower bound is defined to be the maximum lower bound encountered since the beginning of the Evans iteration. Finally, the convergence criterion for Evans algorithm can be defined as follows:

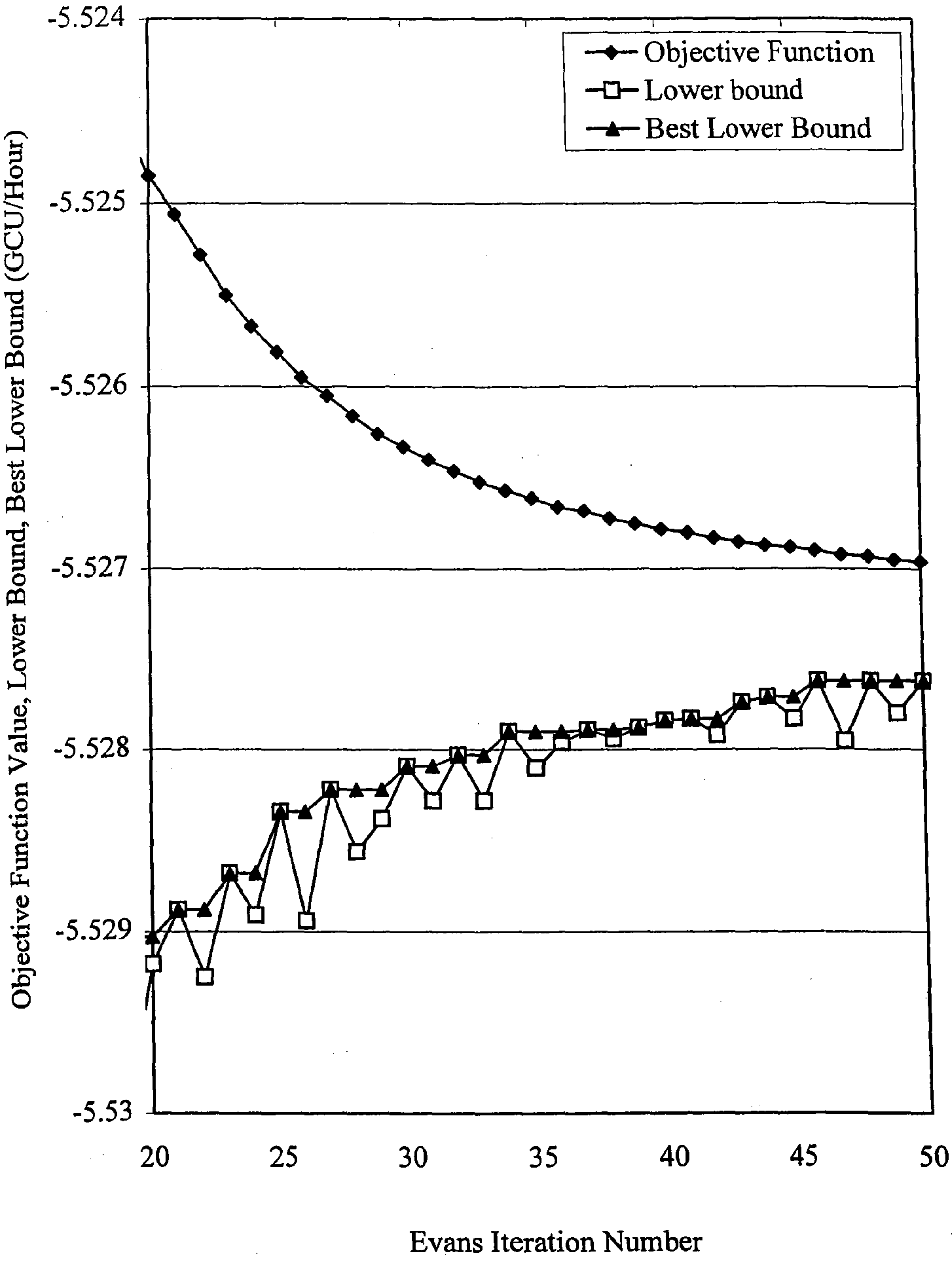


Figure 1: Objective Function values, Lower Bounds and Best Lower Bounds w.r.t. Evans Iterations

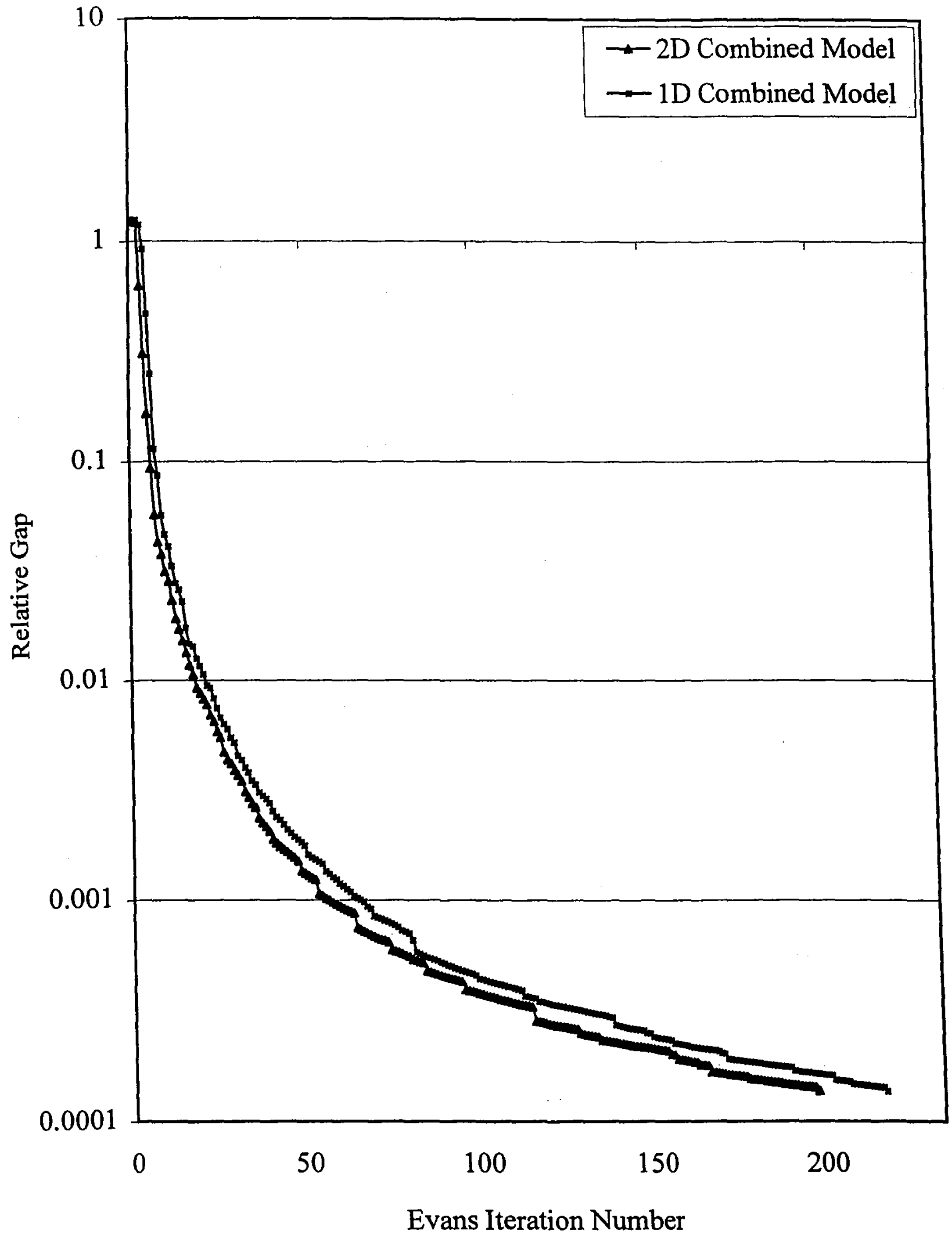


Figure 2: Convergence characteristics of Evans Algorithm

$$\frac{g(\mathbf{v},\ \mathbf{P}) - BLB}{BLB} < \varepsilon_e \tag{2.37}$$

where

ε_e = predefined Evans Accuracy.

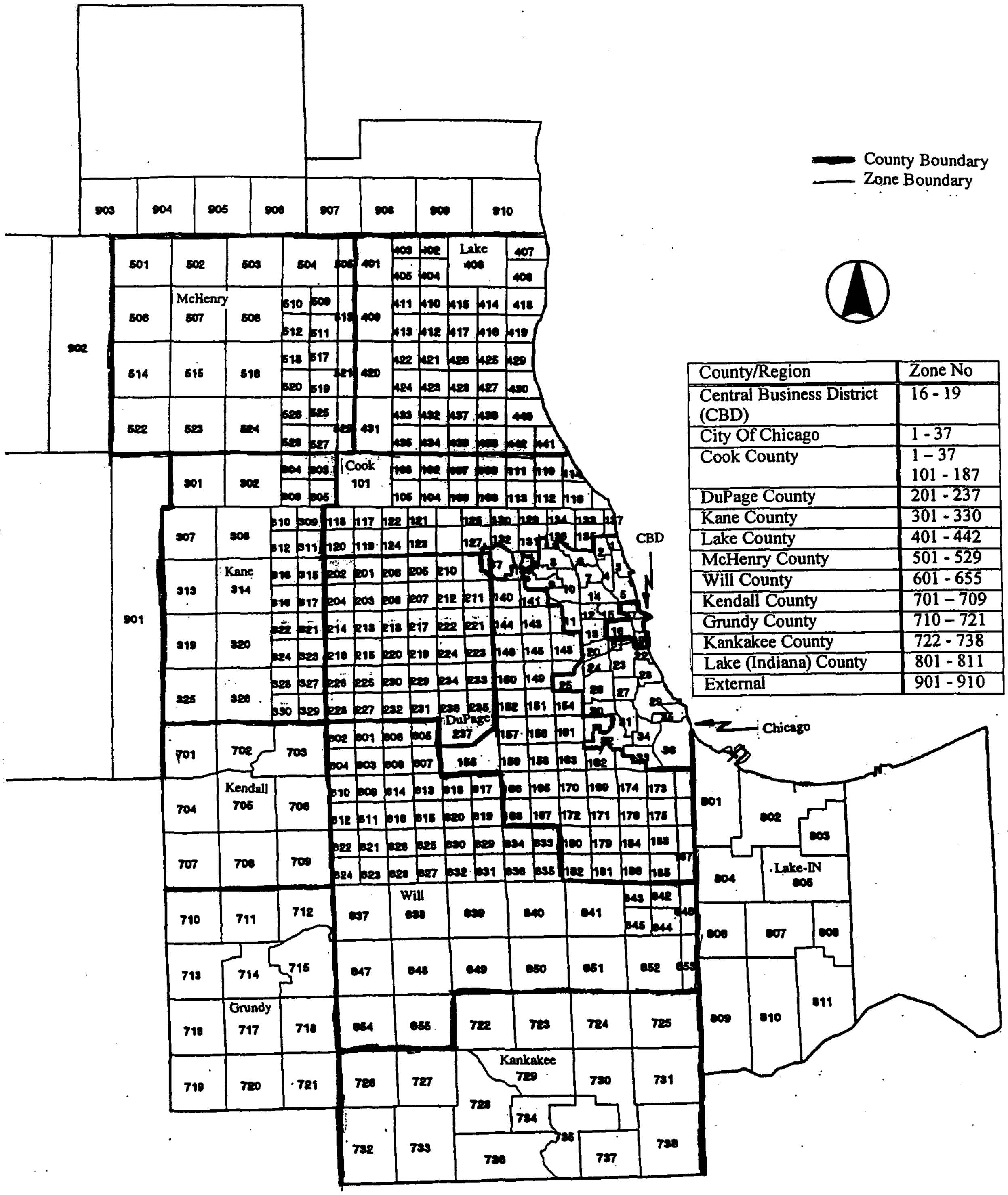
3. VALIDATION OF TRANSIT DATA AND MODEL IMPLEMENTATION

3.1 The Transportation Network

The current study was conducted on a transportation network representing the Chicago metropolitan area, also referred to as the Chicago Region. The Chicago Region consists of ten counties: Cook, DuPage, Grundy, Kane, Kankakee, Kendall, Lake, Lake-Indiana, McHenry and Will (Figure 3). Cook County contains the City of Chicago. The Central Business District (CBD) of the City of Chicago will be called the CBD for the entire metropolitan area in this document.

The region consists of 387 zones (Figure 3). The road network used in this study is named the Sketch Network, and is an aggregation of a larger network covering the same region and consists of 933 nodes and 2950 links. The zones in each county are given in Figure 3. Sizes of the zones range from 9 square miles in the interior to 36 square miles in outlying areas.

The computer implementation of the Combined Model used in this research does not employ a transit assignment. Instead, zone-to-zone cost-minimizing transit cost components are inputs to the program. Since transit travel takes place along fixed routes, this approach is convenient. Consequently, the links in the network are all auto network links representing the freeways and arterials in the Chicago metropolitan area.



County/Region	Zone No
Central Business District (CBD)	16 - 19
City Of Chicago	1 - 37
Cook County	1 – 37 101 - 187
DuPage County	201 - 237
Kane County	301 - 330
Lake County	401 - 442
McHenry County	501 - 529
Will County	601 - 655
Kendall County	701 – 709
Grundy County	710 – 721
Kankakee County	722 - 738
Lake (Indiana) County	801 - 811
External	901 - 910

Figure 3: Chicago Metropolitan Area and its zones

(Source: Chicago Area Transportation Study, Chicago, Illinois)

Each link in the network has its own length, free-flow speed, number of lanes and capacity per lane. Although the freeway links represent actual freeway segments, the links representing the arterials do not. Since there are numerous arterials in the area, the network was kept to a reasonable size by aggregating the arterial links. Aggregation of the arterials was done by assigning appropriate number of lanes to the arterial links so that each such link has the same capacity as the group of arterials it represents.

3.2 Transit Service in the Chicago Region

The Chicago metropolitan area is served by a well-developed transit system. Four different transit services operate within the region. These are: Chicago Transit Authority (CTA) buses, CTA trains, Metra commuter trains and Pace buses. CTA buses and trains mainly operate within the City of Chicago and Pace buses primarily serve the suburbs. Metra trains connect the suburbs and the CBD. According to the 1990 Household Travel Survey conducted in this metropolitan area (Appendix A), the use of Pace buses is extremely low, as shown in Figure 4. CTA bus routes and train lines are rather densely distributed within the City of Chicago, whereas Metra lines extend radially outwards from the CBD. Within the City of Chicago the use of Metra is minimal, as shown in Figure 5, whereas in the rest of the Chicago region the predominant transit service is Metra commuter rail, as shown in Figure 6. Figure 7 shows that in 1990 there were ten Metra lines extending in various radial directions. For this study the Metra lines were numbered from 1 to 10 as shown in Figure 7.

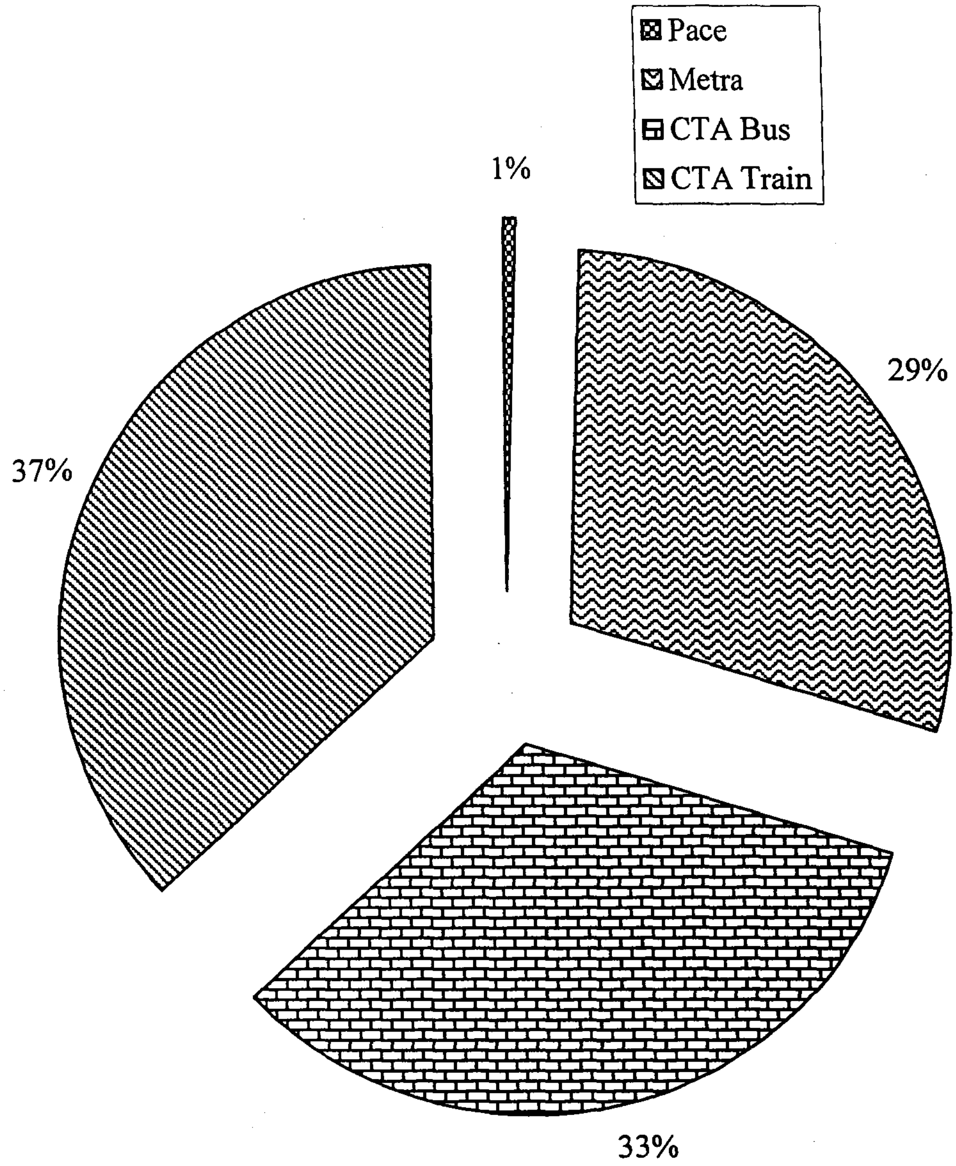


Figure 4: Relative use of transit services in the Chicago Region

(Data source: Household Travel Survey, Chicago, Illinois, 1990)

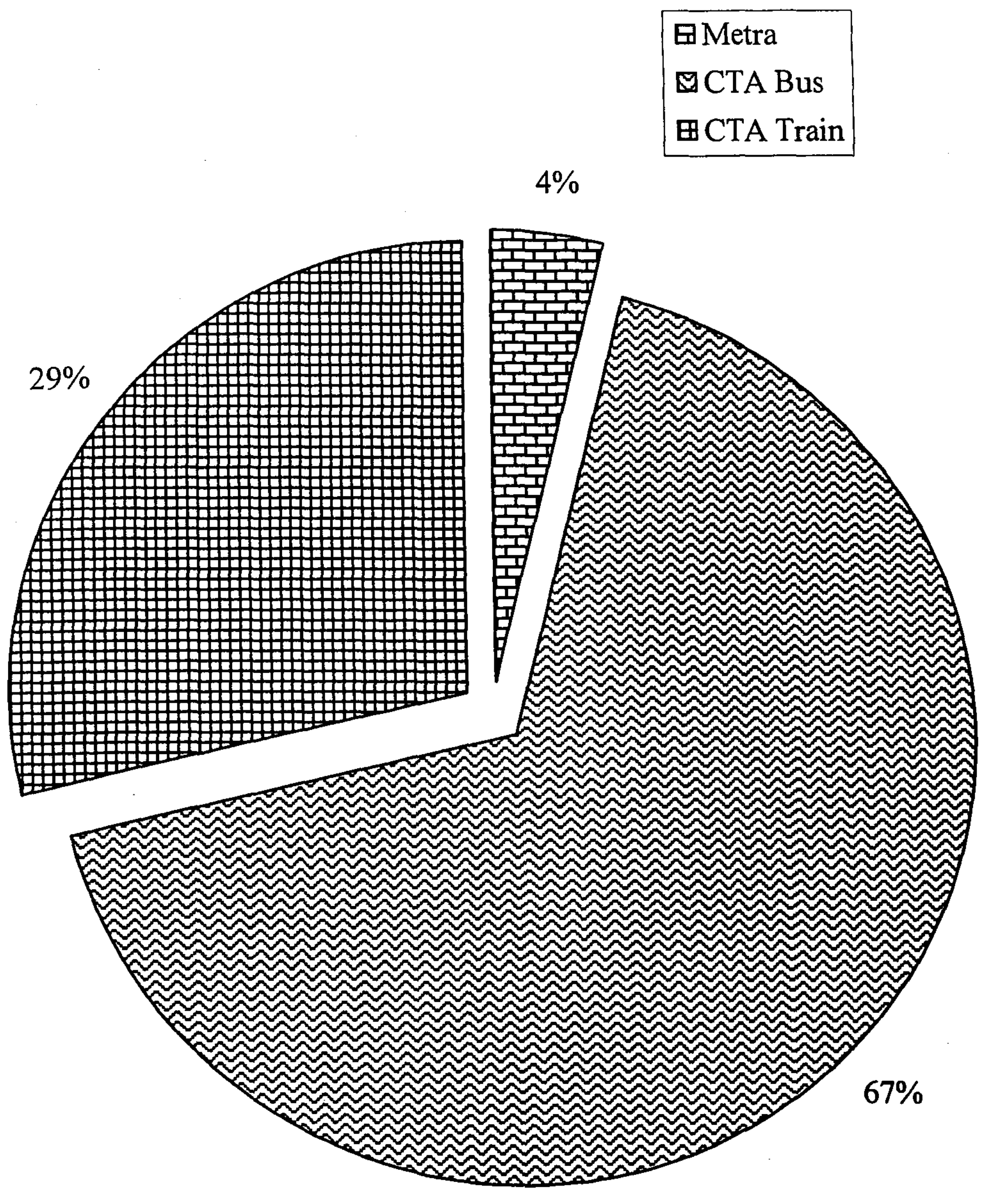


Figure 5: Relative use of transit services within the City of Chicago

(Data source: Household Travel Survey, 1990, Chicago, Illinois)

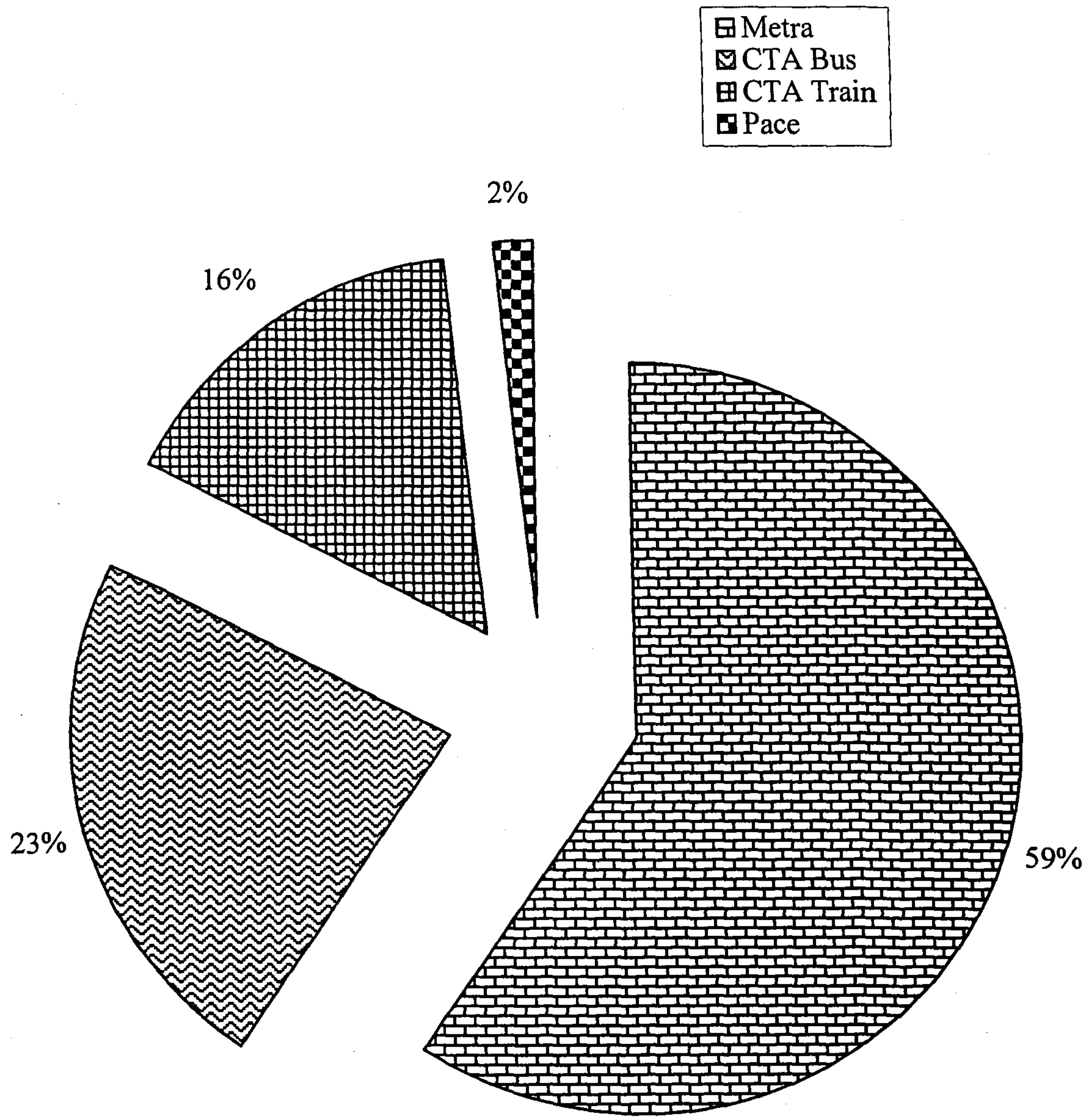


Figure 6: Relative use of Transit services outside the City of Chicago in the Chicago Region

(Data source: Household Travel Survey, 1990, Chicago, Illinois)

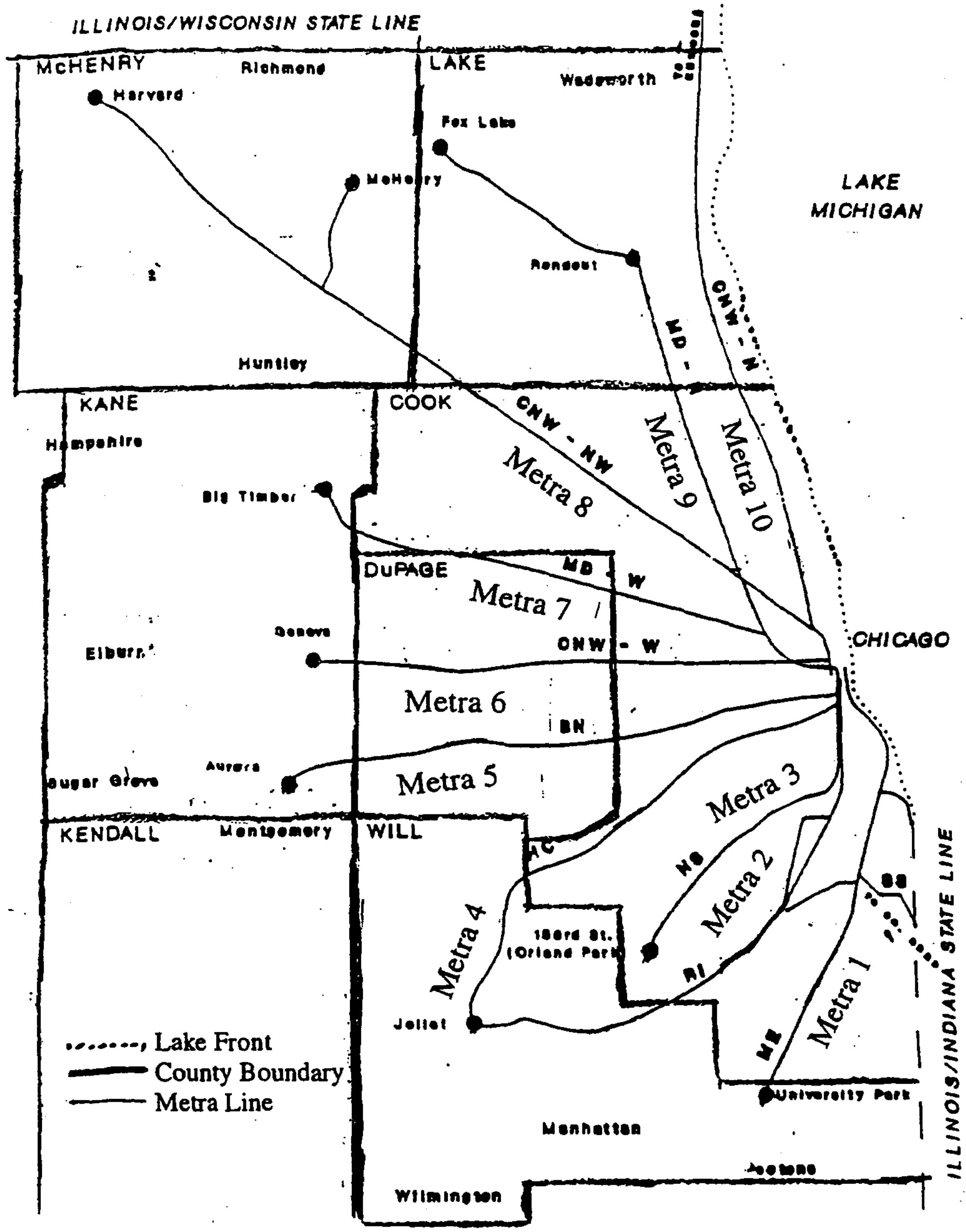


Figure 7: Metra Lines in the Chicago Region in 1990

(Source: Regional Transportation Authority, Chicago, Illinois)

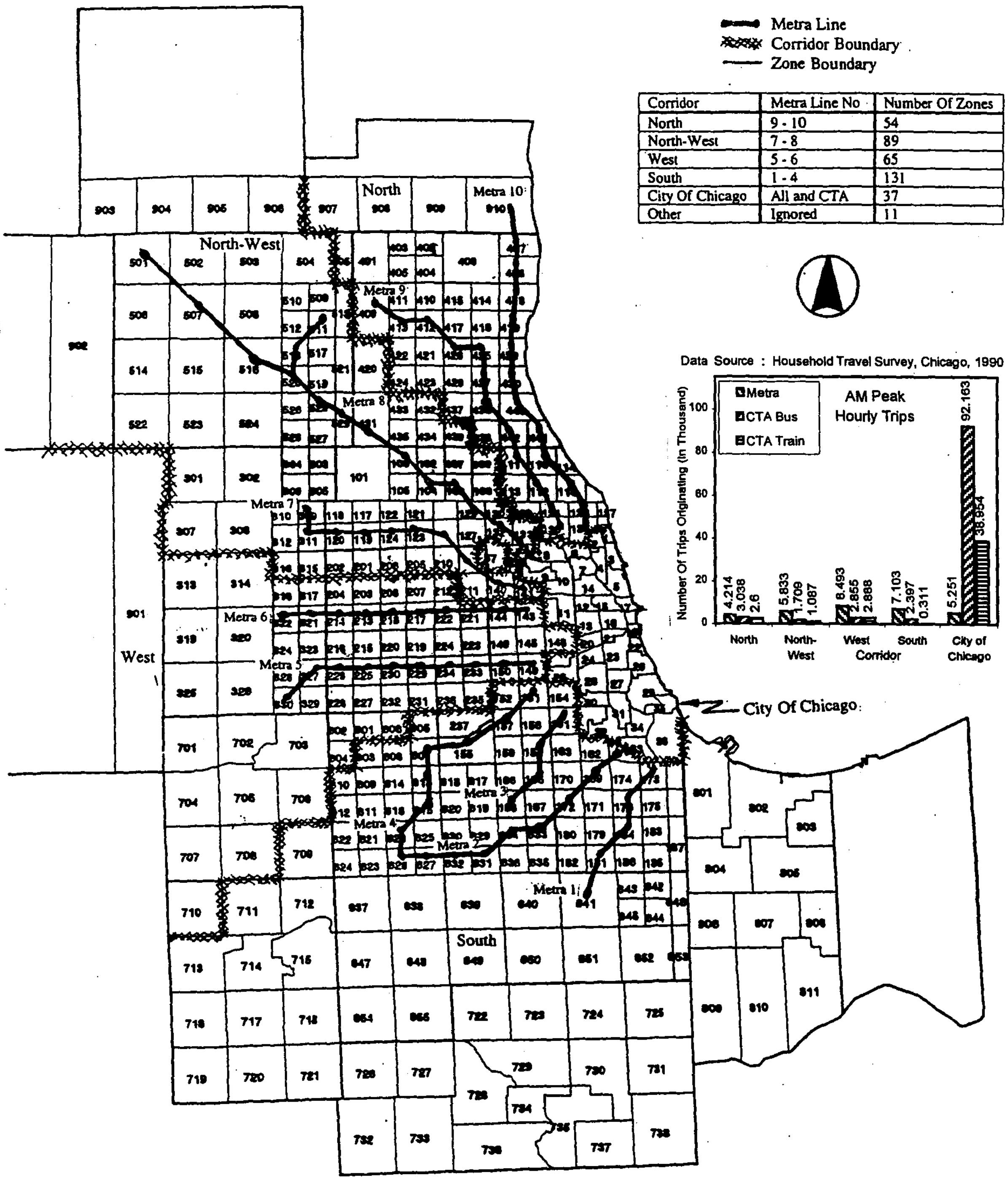
3.3 Transit Corridors

Because of the radial layout of the Metra lines, it is possible to logically divide the region into several radial corridors. Each corridor can be regarded as the area served by a group of Metra lines. However, since all four services have operations to some extent within the City of Chicago, and moreover, since the City of Chicago is densely crossed with CTA trains and buses, there is interaction among all these services. Without a precise knowledge of boarding, or more generally speaking by performing a transit assignment, it is unlikely any model can capture these interactions. In such a situation, the City of Chicago can be considered to be a separate corridor itself. For this study the zones were divided into five corridors, namely North, North-West, West, South and Chicago. Figure 8 shows the corridor layout and the Metra lines and zones associated with each corridor.

3.4 Origins and Destinations

One main input to the Combined Model is the estimated totals of the trips entering and leaving the individual zones in the network. The total numbers of trips leaving zones are known as Origins and those entering the zones are called Destinations.

The Combined Model combines the origin-destination choice, mode choice and trip assignment parts of the travel forecasting procedure. Yet another part, trip generation, is external to the Combined Model. Trip generation models estimate these origins and destinations.



Corridor	Metra Line No	Number Of Zones
North	9 - 10	54
North-West	7 - 8	89
West	5 - 6	65
South	1 - 4	131
City Of Chicago	All and CTA	37
Other	Ignored	11

Figure 8: Transit Corridors in the Chicago Region

The origins and destinations used in this study were prepared by using productions and attractions estimates obtained from CATS. Productions and attractions are a condensed representation of origins and destinations. Depending on the activities at the trip ends the definitions of productions and attractions differ. For trips between home and some other activity like work, shopping etc., a trip made from home to work and another made from work to home, are jointly termed as two productions at the home end and two attractions at the work end. On the other hand for trips made from places other than home productions and attractions are the same as the origins and destinations respectively.

CATS used its own trip generation model, which is essentially a linear regression model, to calculate productions and attractions. The data consisted of home-based work, home-based shop, home-based other and non-home based productions and attractions. The non-home based productions and attractions directly translate to origins and destinations respectively. However, in order to convert the other types of productions and attractions, the relationship between those and the corresponding type of origins and destinations must be found. Household Travel Survey was used for this purpose. The procedure used to convert productions and attractions into origins and destinations was empirical; an example will explain the method better.

For the home based work type, Household Travel Survey trips were used to prepare a home-to-work and a work-to-home trip matrix. The sum of the first and the inverse of the second matrix give the home based work trip matrix. Row sums of these individual matrices produced home-to-work origins, work-to-home origins and home based work productions respectively. Similarly, column sums gave the home-to-work destinations, work-to-home destinations and home based work attractions. These observed sets of origins, destinations, productions and attractions gave information such as, what fraction of the home based productions are home-to-work origins the fraction of the home based work attractions are home-to-work destinations etc. Applying these same factors to CATS trip generation home based work productions and attractions home-to-work origins and destinations, and work-to-home origins and destinations were obtained. A similar procedure was followed for the home based shopping and home based other productions and attractions.

Finally all the sets of origins and destinations obtained from the CATS productions and attractions estimates were summed in order to obtain a single set of total origins and destinations. However, it is customary that the sum of the origins are equal to the sum of the destinations, implying that the number of trips originating in the region during the analysis period is equal to the number of trips ending within the region during the same period of time. But the total origins and destinations produced so far were not in accordance with this origin-destination equilibrium and needed to be modified.

In order to modify the origins and destinations for bringing their sum to the same number, the sum of the origins and the destinations were calculated. The difference between these two sums was halved giving the quantity of total adjustment needed for the origins and destinations. In order to adjust the origins, the total amount of adjustment was evenly distributed to all the origins. This was done by addition if the sum of the origins was below that of the destinations or by subtraction otherwise. The destinations were adjusted using the total adjustment in a similar fashion. The final origins and destinations had about 60% work related trips; the remainder were non-work related trips.

3.5 Parameters Used in the Study

The most important parameters needed to solve the Combined Model are the cost parameters that are used to calculate generalized travel cost (Equations 2.13 and 2.16). The values of these and the other parameters used in this research are shown in Table II. Transit bias is not included in this list, because transit bias was calculated with a separate analysis explained later.

The auto occupancy factor was prepared using the 1990 Household Travel Survey data. Other parameters came from a separate parameter estimation analysis performed by Xin Tian (1999). Parameter estimation involves the solution of a bi-level optimization model. The lower level optimization part is the Combined Model and the upper level maximizes the likelihood of the trip matrix produced by the lower level being close to some observed trip matrix in terms of the mean origin-destination flow. Parameter estimation was not a part of this work and a detailed description of the model used is beyond the scope of this thesis.

TABLE II

COMBINED MODEL PARAMETER VALUES USED IN THE STUDY

Parameter Name	Symbol	Value	Unit
Cost Parameter for Auto IVT	γ_1	0.16600	GCU/Minute
Cost Parameter for Auto OVT	γ_2	0.24500	GCU/Minute
Cost Parameter for Auto Operating Cost	γ_3	0.00163	GCU/Cent
Cost Parameter for Transit IVT	γ_4	0.03400	GCU/Minute
Cost Parameter for Transit OVT	γ_5	0.12900	GCU/Minute
Cost Parameter for Transit Operating Cost	γ_6	0.01690	GCU/Cent
Cost Sensitivity Parameter	μ	1.00000	1/GCU
Auto Occupancy Factor	H	1.14000	Persons/Vehicle

3.6 Link Cost Function

In the formulation of the Combined Model used in this work, there are two classes of flows involved: origin-destination flows and link flows. Origin-destination flows are computed from route flows and route flows are converted into link flows (Equation 2.6). One of the components of travel cost is in-vehicle travel time (IVT). Route IVT depends on the route of the trip and for each route it is an accumulation of the IVT on all the links comprising the route. In the case of auto, link IVT is an increasing function of link flow in order to incorporate the effect of congestion on auto travel. However, in order to take into account the role of link capacity in congestion phenomenon, the link IVT for auto must also be a function of link capacity. The link IVT function, more commonly known as the Link Cost Function, must be such that the IVT should be increasing. However, the rate of increase at low level of link flow should be slow, and at a level of link flow near or above link capacity should be very high. There are several link cost functions; the most popular one was proposed by the Bureau of Public Roads (BPR), which is the one used in this study as given below.

$$c_a(v_a) = F_a\left(1+0.15\left(\frac{v_a}{K_a}\right)^4\right) \tag{3.1}$$

Where,

$c_a(.)$	=	IVT on link a (minute)
v_a	=	flow on link a (vehicles/hour)
F_a	=	free-flow travel time i.e. IVT at zero flow on link a (minute)
K_a	=	capacity of link a (vehicles/hour)

3.7 Analysis Period

Transportation networks are usually analyzed for the flow of traffic over a certain period of time, referred to as the Analysis Period. A two-hour morning peak period is adopted in this research. More specifically all the trip-related input to the model and output from the model were the average hourly quantities during a typical week-day morning peak period. Since this study concentrates on an issue related to transit trips originating in different transit corridors, the CBD is an important destination for transit trips, and the heaviest volume of CBD-bound transit traffic is observed during the morning peak period, this period was selected. The one-hour duration of the analysis period is just a convenient convention adopted in this work.

In order to identify the morning peak period, trips reported in the 1990 Household Travel Survey were analyzed. Household Travel Survey trips were made on typical weekdays and the start and end times of the trip were reported in 24 hour format. For the peak period analysis, the 24 hour period was divided into 15 minutes intervals. These intervals were marked as follows. The hours were denoted by a number from 0 to 23, 0 meaning the first hour starting at 12:00 midnight of the survey date and 23 implying the last hour starting at 11:00 PM on the survey date. Each hour has four 15 minutes intervals, time-sequentially assigned from 1 to 4. For example interval 7.1 is the period from 7:00 AM to 7:15 AM and the interval 15.4 spans from 3:45 PM to 4:00 PM. Figure 9 shows the distribution of the start time of transit trips in the Chicago region over a typical weekday. Figure 10 is the corresponding distribution for auto trips. The period from 6:45 AM to 8:45 AM was chosen as the morning peak period after reviewing Figures 9 and 10. All trip information input to and obtained from the model was average hourly data over this period.

3.8 Combined Model Implementation

The implementation of the Combined Model used in this study, after appropriate enhancement, was developed by the TransLab at the University of Illinois at Chicago

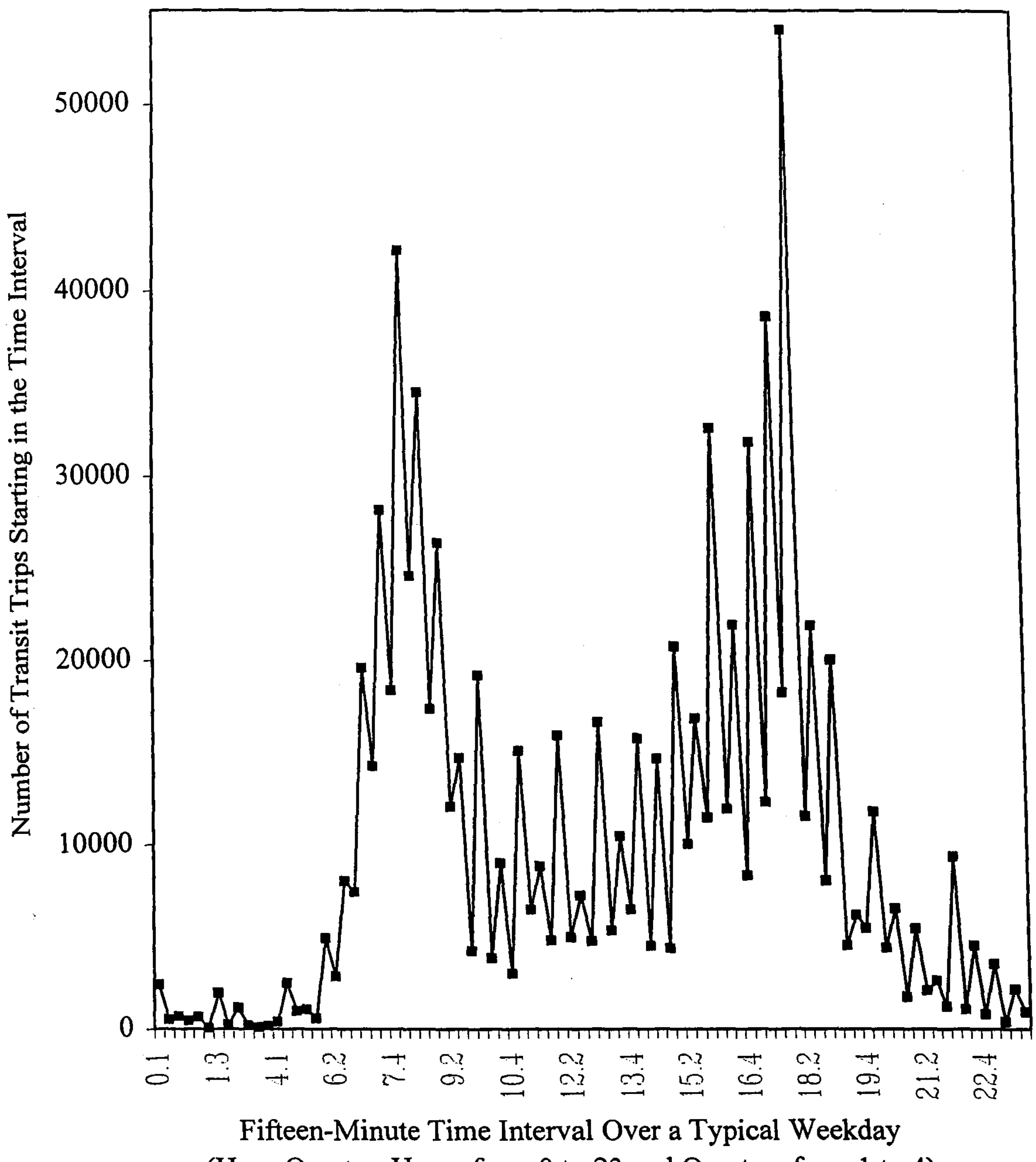


Figure 9: Start time distribution of transit trips in the Chicago Region in a typical weekday

(Data source: Household Travel Survey, Chicago, Illinois, 1990)

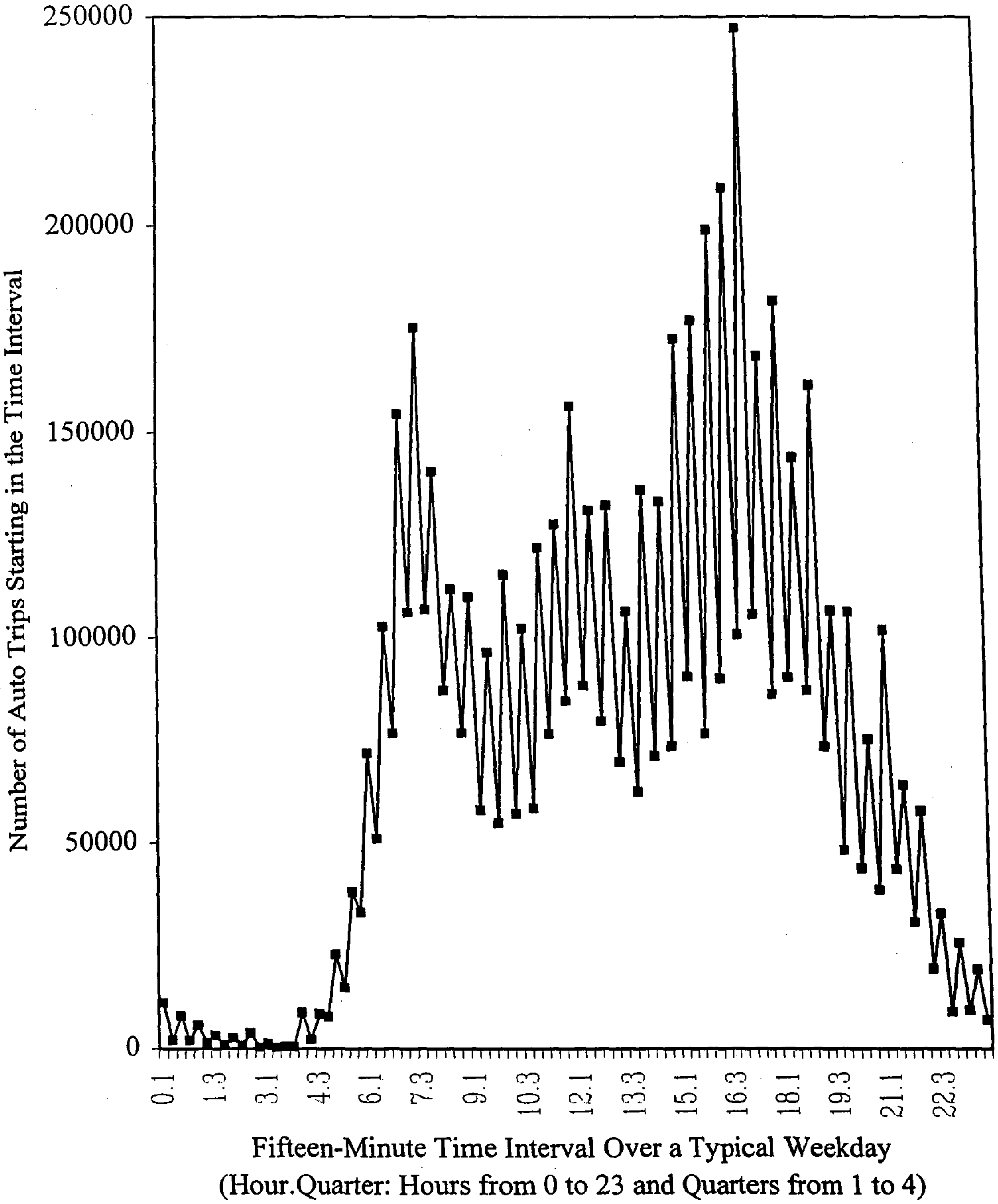


Figure 10: Start time distribution of auto trips in the Chicago Region in a typical weekday

(Data source: Household Travel Survey, Chicago, Illinois, 1990)

(UIC). The computer implementation was done using the C language. The program is solved on both Personal Computers (PCs) and on Unix machines and manages its memory requirement dynamically. Both single precision arithmetic and double precision arithmetic versions of the program are available. The double precision PC version of the code was used in this study. After solving the model, the program gives many summary results, based on the zone-to-zone trip matrices by different modes that the model predicts, along with other output such as the trip matrices. The summary output, also called the Summary Measures, helps to evaluate model's performance quickly and compare different solutions easily. Some of the summary averages are: Vehicle Miles Traveled (VMT), Person Miles Traveled (PMT), Vehicle Hours Traveled (VHT), Person Hours Traveled (PHT), auto operating cost, transit fare, OVT, IVT etc.

3.9 Modification to the Combined Model Implementation

The 2D Combined Model was implemented as a C program. New trip matrix balancing modules were added to this program so that the 1D Combined Model could also be solved. The code was further enhanced to output the origin and destination attractiveness factors for all the network zones after solving the 2D Combined Model. This was necessary because the 1D Combined Model needs estimates of destination attractiveness factors as input; those obtained from a 2D Combined Model solution were used for this purpose. This issue of attractiveness factor feedback will be discussed later.

The central issue of this work was the transit ridership in different corridors with regard to transit line extension and model formulation. As a result calculation of transit rider in different corridors was crucial to this research. In order to calculate transit ridership, additional input to the model was necessary consisting of the zone-to-corridor mapping information. Each transit corridor was assigned a Corridor Number and each zone was given an attribute, which was the number of the corridor to which the zone belongs. A new module was added to the C code to use this mapping data and the transit trip matrix produced by the model to calculate the total number of transit trips originating in different corridors and going to different destinations.

3.10 Transit Cost Component Data

Transit cost component data consists of the IVT, OVT and fare associated with transit trips between every pair of zones in the network. OVT includes the access time to the origin transit station, waiting time, transfer time between different services and egress time from destination transit station. The access mode can be either auto, bus or walk. The egress mode is generally walk. If it is not possible to travel from one zone to another zone by transit, all the cost components were set to zero for that origin-destination pair, indicating no service. Since the zones are fairly large, even intra-zonal transit trips have IVT, OVT and fare associated with them.

Since transit data is used in the implementation of this Combined Model as a substitute of transit assignment, in order to be in congruence with the cost minimizing objective of the model, these data must also be cost minimizing. Therefore, in preparing the transit data for every pair of zones, the choice of transit service, initial and final stations, sequence and service of transfer, access mode etc. must produce minimum generalized travel cost by transit. More specifically, considering all the available options and their possible combinations, when the overall transit travel plan is fixed between a zone pair, the resulting generalized cost of transit travel between that origin-destination pair should be the minimum possible.

Two sets of transit data were available for this study. For the sake of convenient description these sets will be called CATS Transit Data and TransLab Transit Data. Detail about each of these data sets follows.

<u>CATS Transit Data</u>: This set of transit data was prepared by the Chicago Area Transportation Study (CATS) for the Sketch Network, which was used in this research.

<u>TransLab Transit Data</u>: This set was obtained by converting yet another set of transit data, which was prepared in the TransLab at UIC and is called the Regional Transit Data subsequently. The Regional Transit Data were prepared for a larger network encompassing the same region as the Sketch Network i.e. the Chicago metropolitan area, known as the Regional Network. Regional Network has 1,790 zones, 12,982 nodes and 39,018 links. Since both the Sketch and the Regional Network covers the same area there is unique

correspondence between the Sketch Network zone and the Regional Network zones. Each Sketch Network zone is comprised of several Regional Network zones. However, since the zone boundaries in the two networks do not coincide, it is possible that a Regional Network zone might not be entirely within only one Sketch Network zone. In that case a factor, which can be called a Contribution Factor, can be generated, which would indicate what fraction a Regional Network zone contributes to a Sketch Network zone. Obviously, the values of such contribution factors range from 0 to 1. If any zone in the group of Regional Network zones comprising a Sketch Network zone has a contribution factor equal to zero this implies that this Regional Network zone is completely outside of the boundaries of that Sketch Network zone and hence can be excluded from the group. On the other hand a contribution factor value of 1 in such a case would mean that the Regional Network zone lies entirely inside the Sketch Network zone. This policy results in Regional Network zone to Sketch Network zone mapping information. This correspondence data was used to convert the Regional transit data into TransLab transit data according the following formulae.

$$\tau^{\sigma}_{i_\sigma j_\sigma} = \frac{\sum_{i_\rho \in I^{\rho}_{i_\sigma}} \sum_{j_\rho \in J^{\rho}_{j_\sigma}} A^{i_\sigma}_{i_\rho} A^{j_\sigma}_{j_\rho} \tau^{\rho}_{i_\rho j_\rho}}{\sum_{i_\rho \in I^{\rho}_{i_\sigma}} \sum_{j_\rho \in J^{\rho}_{j_\sigma}} A^{i_\sigma}_{i_\rho} A^{j_\sigma}_{j_\rho}} \tag{3.2}$$

Where,

i_σ = Sketch Network trip origin zone

j_σ = Sketch Network trip destination zone

i_ρ = Regional Network trip origin zone

j_ρ = Regional Network trip destination zone

$\tau^{\sigma}_{i_\sigma j_\sigma}$ = Sketch Network transit cost component from Sketch Network zone i_σ to Sketch Network zone j_σ

$\tau^{\rho}_{i_\rho j_\rho}$ = Regional Network transit cost component (the same type of component as $\tau^{\sigma}_{i_\sigma j_\sigma}$) from Regional Network zone i_ρ to Regional Network zone j_ρ

$I^{\rho}_{i_\sigma}$ = set of Regional Network zones composing the Sketch Network zone i_σ

$J^{\rho}_{j_\sigma}$ = set of Regional Network zones composing the Sketch Network zone j_σ

$A^{i_\sigma}_{i_\rho}$ = contribution factor for Regional Network zone i_ρ w.r.t. Sketch Network zone i_σ

$A^{j_\sigma}_{j_\rho}$ = contribution factor for Regional Network zone j_ρ w.r.t. Sketch Network zone j_σ.

Preparation of transit data has different, rather empirical, approaches. CATS and Regional transit data were obtained as data for this project and the detailed discussion of their preparation methodology is beyond the scope of this thesis. However, the procedure involved is outlined in Appendix B.

3.11 Transit Data Validation

Given the availability of two sets of transit data a decision had to be made about which one to use. In order to resolve the issue, a comparative study was conducted to evaluate the relative performance of the two data sets with regard to the Combined Model and to explore the characteristics of the data. Although the solution of the combined model produces zone-to-zone travel demand by modes, comparison of this unprocessed output was unlikely to provide any insight into the data. Instead, evaluation of the transit data was partly based on the summary output, mentioned in section 3.7, produced by the program implementing the combined model.

One very important criterion in the validation of the transit data was the value of the transit bias parameter (γ_7). As explained in Chapter 2, transit bias is a component of generalized cost of travel by transit and an indication of the relative preferences that travelers have for transit compared to auto. Transit bias is supplied to the Combined Model as an input. One interesting function of transit bias in the solution of the Combined Model is that it controls the overall Transit Share, i.e. the fraction of total trips taking transit. Consequently, it is possible to assign a certain value to this parameter so that the overall transit share predicted by the model equals an observed value. This particular value of transit bias can be obtained by trial-and-error or by a method of interpolation.

Forcing the model to predict a predefined transit share is also equivalent to adding another constraint to the Combined Model. In fact, the Combined Model can be formulated to omit the transit bias term from the objective function and include a transit share constraint, which is given below.

$$\sum_i \sum_j P_{ijt} = \overline{P}_t \qquad (3.3)$$

where

i	=	origin zone
j	=	destination zone
P_{ijt}	=	proportion of transit trip from zone i to zone j
$\overline{P}_t$	=	observed proportion of total trips made by transit over the entire network.

Formulation of such a model is described in Appendix C and will be referred to later as the 3D Combined Model. In fact, a computer program to solve the 3D Combined Model was developed at TransLab. From the trip matrices produced by the solution of 3D Combined Model it is possible to compute the average transit bias. If the 3D Combined Model is solved by setting $\overline{P}_t$ equal to the observed transit share, use of the resulting transit bias in the 2D and 1D Combined Model produces the observed transit share in the predictions of those models too. This approach was adopted in this study for finding the transit bias value. The value of the observed transit share used was 0.16, which was obtained from the Household Travel Survey.

However, the use of transit bias in the 1D and 2D Combined Model introduces an exogenous, and rather artificial, control on the behavior of the models. The higher the absolute value of the transit bias the higher is the degree of this forced behavioral control. Obviously a lower absolute value of transit bias is more desirable.

Table III enumerates the transit bias from the 3D Combined Model and the summary results obtained from the trip matrices predicted by the 2D Combined Model, for the two sets of transit data. It can be seen that both data sets predict virtually the same level of auto cost and auto use. However, with the TransLab data although transit cost components are significantly lower, transit PMT is practically the same as that with the CATS data and the to-CBD Transit Share is lower. Here to-CBD transit share is the fraction of the CBD bound trips made by transit. The most important observation in Table III is the difference in the transit bias value with the two transit data sets. The absolute value of the transit bias needed to produce a 16% overall transit share with the TransLab data is 77% lower than that with the CATS data. As mentioned earlier, a lower absolute value of transit bias is always preferable; hence this observation is a very appealing point in favor of the TransLab transit data.

TABLE III

COMPARISON OF THE COMBINED MODEL RESULTS FOR DIFFERENT TRANSIT INPUT DATA

Item	Unit	Combined Model	CATS Transit Data	TransLab Transit Data	Change w.r.t. CATS Data (%)
Transit Bias	GCU	3D	-1.408	-0.328	-76.70
Average Auto VMT	Vehicle-Mile	2D	9.366	9.403	0.40
Average Transit PMT	Person-Mile	2D	7.643	7.672	0.38
Average Auto Operating Cost	Cent	2D	117.230	118.164	0.80
Average Transit Fare	Cent	2D	176.905	139.816	-20.97
Average Auto OVT	Minute	2D	6.140	6.151	0.18
Average Transit OVT	Minute	2D	18.162	16.564	-8.80
Average Auto IVT	Minute	2D	12.010	12.107	0.81
Average Transit IVT	Minute	2D	26.460	22.115	-16.42
To-CBD Transit Share	N/A	2D	0.662	0.646	-2.42

As shown in Table III, with both the data sets the value of transit bias is negative. Observation of Equation (2.16), which shows the use of transit bias in the calculation of the generalized cost of transit travel, reveals that negative transit bias can be interpreted as a subsidy of transit use as perceived by the travelers. The lower the absolute value of negative transit bias, the lesser is such user-subsidy associated with transit trips.

Before the final decision was made as to which transit data would be used, the data sets themselves were compared. The transit data histograms presented in Figure 11 through Figure 13 show that the two transit data sets are distinctively different. The CATS data covers only about one-half of the zone pairs in the network; the TransLab data provides transit service between almost two-third of the zone pairs. Although there are many zones in the network that are not directly served by transit, auto or walk access to transit stations and transfer between different transit services help to widen the coverage in practice. Clearly, the wider the range of transit coverage in a transit data set, the more preferable are the data. So, considering the extent of transit service in the network, the TransLab data is superior to the CATS transit data.

After careful review of the available transit data sets, as discussed in this section, it was decide that the TransLab transit data would be used in the rest of the research.

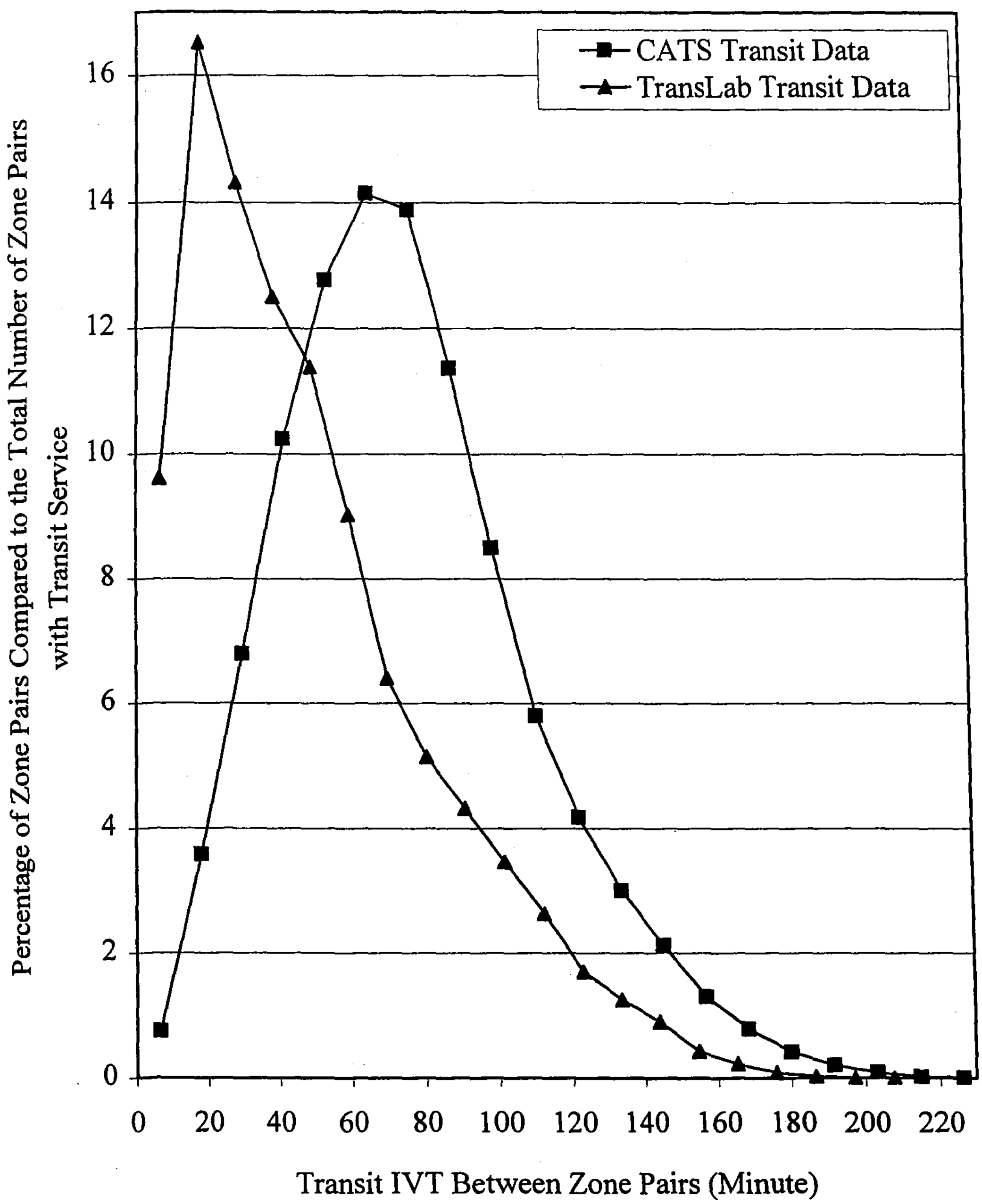


Figure 11: Distribution of zone pairs over IVT from different transit data sets

(CATS transit data: 49.9% zone pairs with no service and 75.06 minutes average IVT; TransLab transit data: 37.9% zone pairs with no service and 48.42 minutes average IVT)

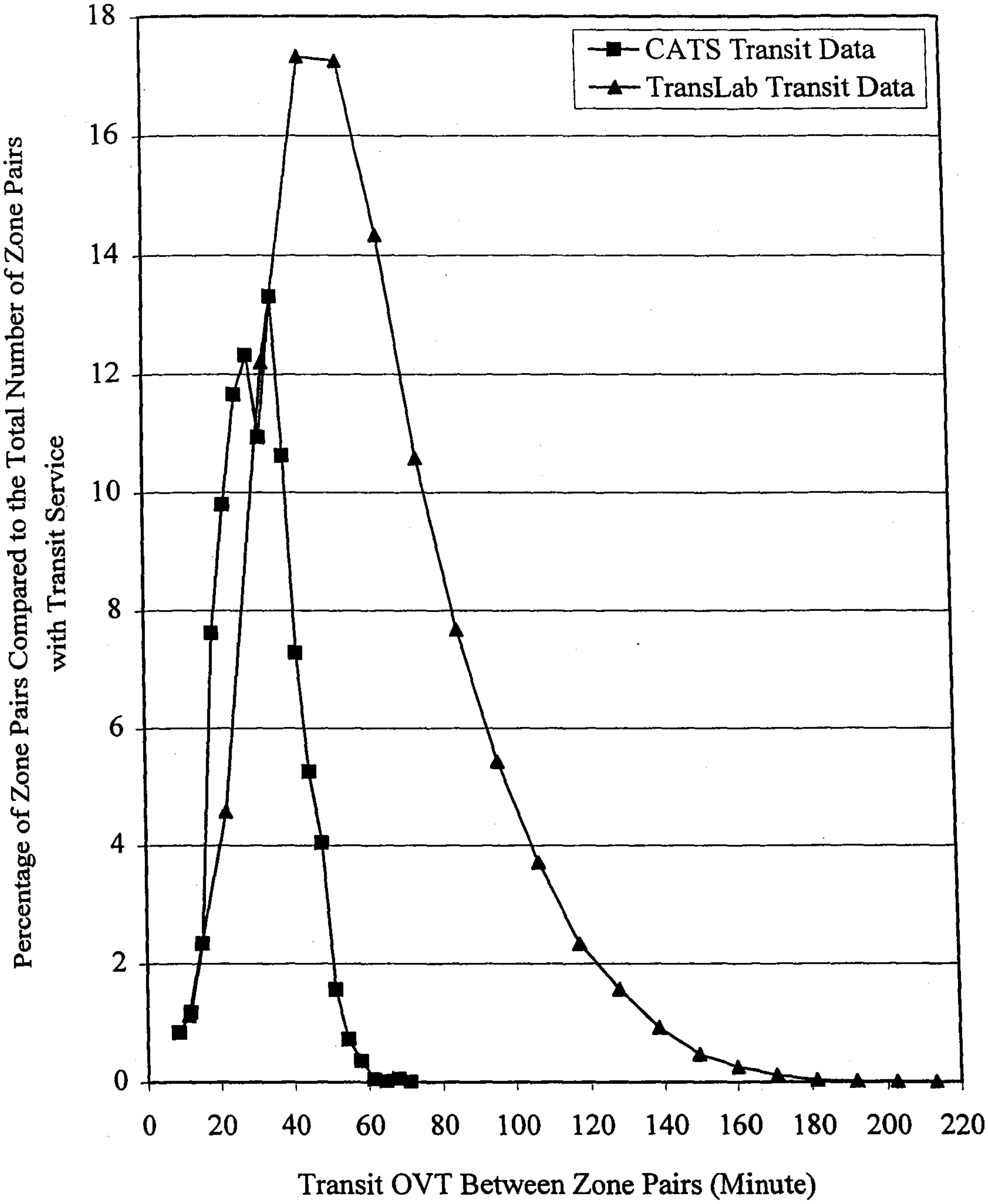


Figure 12: Distribution of zone pairs over OVT from different transit data sets

(CATS transit data: 49.9% zone pairs with no service and 31.51 minutes average OVT; TransLab transit data: 37.9% zone pairs with no service and 62.28 minutes average OVT)

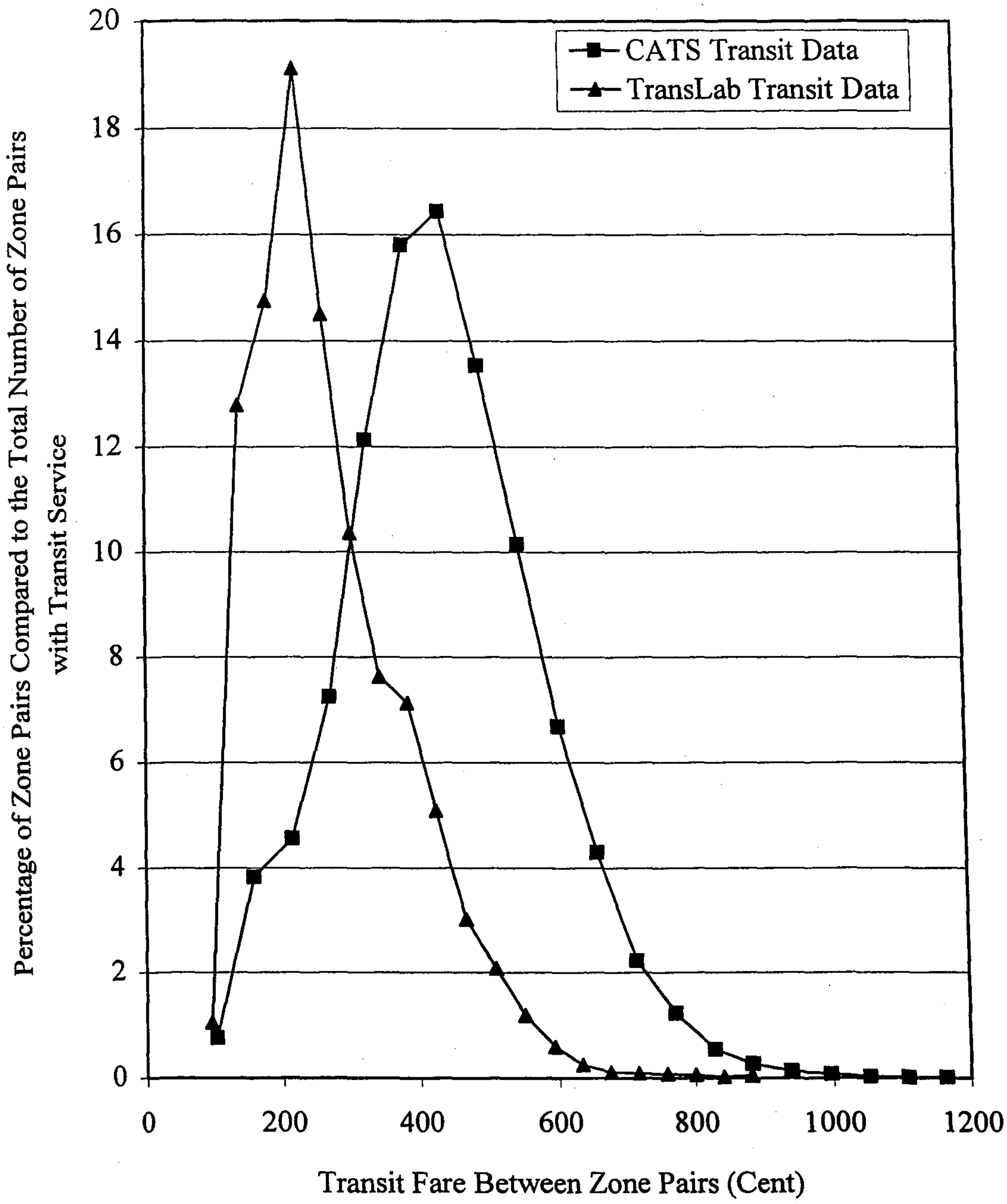


Figure 13: Distribution of zone pairs over fare from different transit data sets

(CATS transit data: 49.9% zone pairs with no service and 435.1 cents average fare; TransLab transit data: 37.9% zone pairs with no service and 271.6 cents average fare)

3.12 Validation of the Implementation of the Solution Algorithm of the Models

Before the 1D and 2D Combined Models can be used to examine the interaction of the origin constraints and destination constraints with transit service extensions, the computer program solving these models must be scrutinized to determine how the implementations of the solution algorithms of the models are performing. This can be done by comparing the results produced by the program after solving the 1D and 2D Combined Models.

As explained earlier, the 1D Combined Model needs estimated destination attractive factors as input. After solving the 2D Combined Model, a set of destination attractiveness factors can be computed using Equation (2.18). If these values of the destination attractiveness factors are used with the 1D Combined Model and if both the models are solved with the same level of Evans accuracy, the output of the solutions of the 1D and 2D Combined Models should be similar. Although the outputs from these two models in such a situation should be closely comparable, they will not necessarily be the same, even with the same computational precision level. The reason is that while solving the 2D Combined Model the values of the attractiveness factors used to calculate flows are different in different Evans iterations. In contrast, the values are same in all Evans iterations in case of the 1D Combined Model. Although the effect of this difference diminishes gradually as solutions of both models converge, the way and to what degree the trip matrix calculation is affected remains indeterminate. Obviously, the differences in trip matrix prediction spills over to link flows and the generalized travel costs.

However, if the models are solved to a fine level of convergence, good agreement is expected between the predictions from the two models.

Once again, summary results are used as a basis of comparison; the outputs from 1D and 2D Combined Models are shown in Table IV. The models were solved twice, once for an Evans accuracy of 0.0014 and again for an Evans accuracy of 0.0000175.

Table IV shows that the prediction by two models are in close agreement, when solved to the same level of convergence and when the destination attractiveness factors from the 2D Combined Model are input into the 1D Combined Model. It is also observed that the level of agreement improves as the Evans accuracy convergence increases. Clearly, the implementations of the 1D and 2D Combined Models are working correctly and more importantly, consistently.

TABLE IV

COMPARISON OF THE 1D AND 2D COMBINED MODEL OUTPUT FOR DIFFERENT EVANS ACCURACY.

Item	Unit	0.00014 Evans Accuracy		0.0000175 Evans Accuracy	
		2D Combined Model	1D Combined Model	2D Combined Model	1D Combined Model
Average Auto VMT	Vehicle-Mile	9.403	9.405	9.390	9.391
Average Transit PMT	Person-Mile	7.672	7.674	7.674	7.674
Average Auto Operating Cost	Cent	118.164	118.166	118.116	118.118
Average Transit Fare	Cent	139.816	139.826	139.825	139.828
Average Auto OVT	Minute	6.151	6.151	6.151	6.151
Average Transit OVT	Minute	16.564	16.566	16.565	16.566
Average Auto IVT	Minute	12.107	12.107	12.104	12.104
Average Transit IVT	Minute	22.115	22.118	22.118	22.118
To-CBD Transit Share	N/A	0.646	0.646	0.646	0.646

4. INTERACTION BETWEEN ORIGIN-DESTINATION CONSTRAINTS AND TRANSIT SERVICE EXTENSIONS

4.1 Design of Experiment

Since the Evans algorithm is iterative, more Evans iterations, or equivalently a smaller value of Evans accuracy, are always preferable because of the resulting higher convergence. However, although during the initial iterations Evans algorithm converges rapidly, the convergence rate slows down sharply as the iterations proceed because of the road assignment. As a result fine convergence substantially lengthens the time required for the solution (Figure 2). Since many comparisons of results from different combined model solutions need to be made for this study, the selection of a fine level of convergence was essential. On the other hand, the large number of solutions needed to get an insight into the interaction of model formulation with transit line extension dictates that the solution time should be reasonable. After some trial and error, an Evans accuracy of 0.000017 was finally used.

The number of balancing iterations was fixed at 1000, which gave a very fine convergence in the vicinity of 10^{-7}. The reason for fixing the number of balancing iterations, not the balancing accuracy itself, was that the balancing accuracy has a very sensitive relationship with the number of balancing iterations needed to achieve the required accuracy. An infinitesimal change in the balancing accuracy limit would make a huge difference in the number of balancing iterations performed. Hence it was more convenient to put a cap on the number of balancing iterations.

With these levels of Evans and balancing accuracy, the solution time of the 2D Combined Model was around five hours and that of the 1D Combined Model was about one hour on a Personal Computer with the Pentium II® processor.

In order to examine the effect of considering and not considering the destination constraints on the prediction of the change in transit ridership due to transit service extension, a separate set of transit data, in addition to the TransLab transit data, was prepared for the extended transit service. The transit data for the existing transit lines, which is the previously discussed TransLab data, is called the Existing Transit Data from now on. The transit data for the extended transit lines, in order to reflect the effect of transit line extensions on the transit cost components, is referred to as the Extended Transit Data.

The models were solved as follows.

1. Solve the 3D Combined Model with the Existing Transit Data to determine the transit bias value. This value is used in steps 2-5.
2. Solve the 2D Combined Model with the Existing Transit Data. This solution will subsequently referred to as the 2D Existing Transit Solution.

3. Solve the 2D Combined Model with the Extended Transit Data. The resulting solution is called the 2D Extended Transit Solution.
4. Solve the 1D Combined Model with the Existing Transit Data; the solution is called the 1D Existing Transit Solution.
5. Solve the 1D Combined Model with Extended Transit Data; the solution obtained is termed as 1D Extended Transit Solution.

The difference between the 2D Existing and the Extended Transit Solution gives the effect of the transit line extension for the 2D Combined Model, whereas the difference between the 1D Existing and Extended Transit Solution shows the 1D Combined Model's prediction of the effect of transit line extension.

The models were exogenously constrained, by setting the value of the transit bias, to produce an overall transit share of 16%, which was obtained from the Household Travel Survey data. Since two sets of transit data were involved in the experiment, the use of separate transit bias values with different data was an option to be considered. However, since the four solutions had to be compared, the use of the same parameter values, which include the transit bias, was more desirable for the sake of consistency. Moreover, the 3D Combined model analysis yielded -0.327967 and -0.327937 bias values for the Existing and the Extended Transit Data, which have a negligible 0.00915% difference in magnitude. Hence the transit bias calculated for the Existing Transit Data was used in all the solutions.

Solution of the 1D Combined Model requires the destination attractiveness factors as input, whereas they are computed by the solution algorithm in case of the 2D Combined Model. However, since the 1D and 2D Combined Model solutions would be compared, there had to be consistency between the 1D and 2D Combined Model solutions. Consequently the destination attractiveness factors computed in the last Evans iteration of the 2D Existing Transit Solution were used in the 1D Existing Transit Solution. Likewise, destination attractiveness factors used in the 1D Extended Transit Solution were those from the last iteration of the 2D Existing Transit Solution. Selection of the last Evans iteration for this purpose was justified by the fact that the last Evans iteration represents the most converged state of the solution.

The CBD bound transit ridership from different corridors was used as the evaluation parameter because transit ridership was the part of the model prediction most directly affected by the transit network extension. The CBD was chosen as the transit ridership destination for the investigation because 59% of the morning peak transit trips are bound to this destination. Figures 14 shows the CBD and non-CBD transit trips by corridor.

4.2 Implementation of Transit Line Extensions

Although the Chicago region was divided into five transit corridors, a hypothetical transit line extension, called the line extension subsequently, was considered in only one corridor for the purpose of this study. The purpose for doing so was two-fold: simplicity of implementation and singling out the effect of the line extension more distinctively. Of

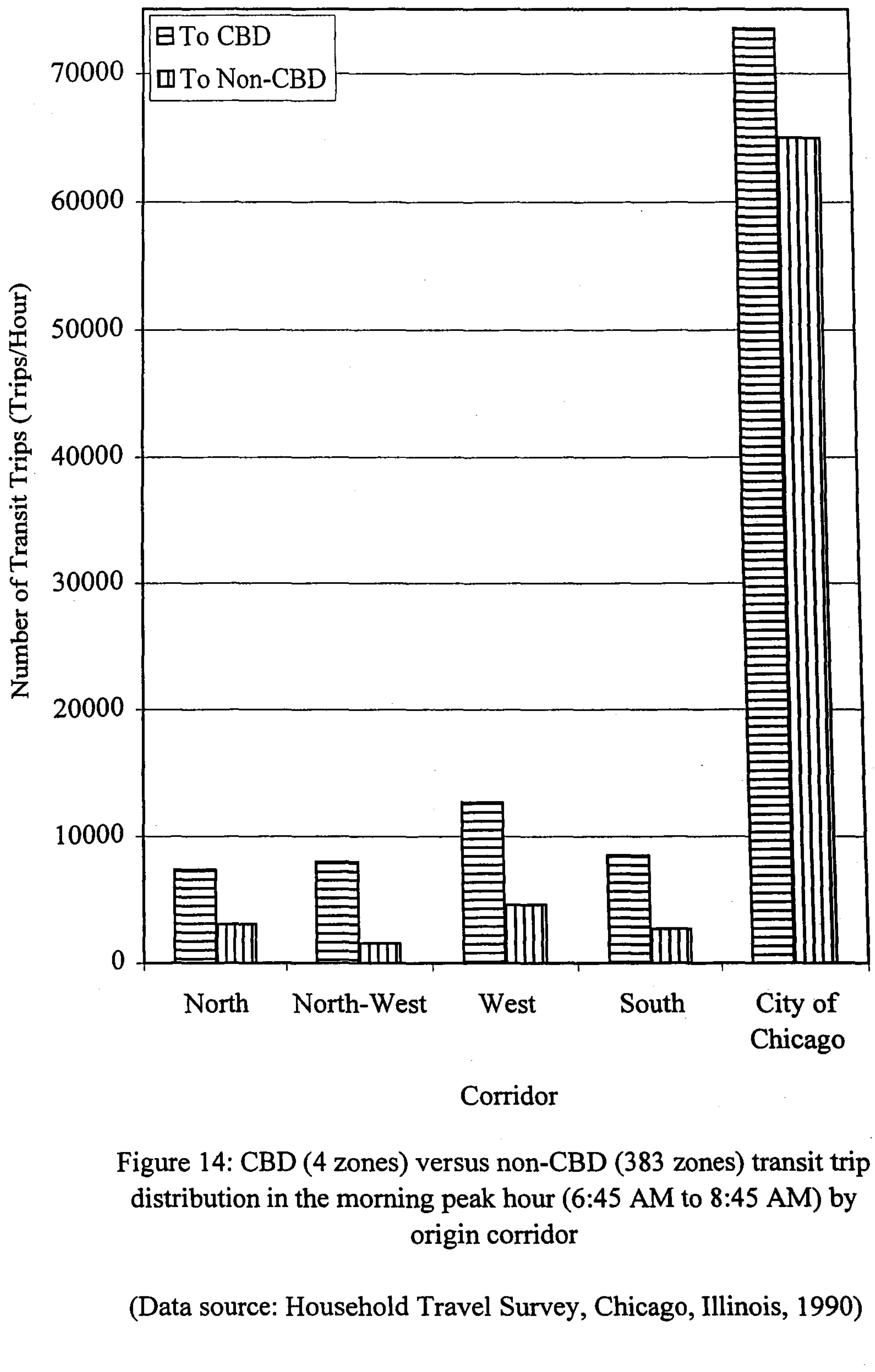


Figure 14: CBD (4 zones) versus non-CBD (383 zones) transit trip distribution in the morning peak hour (6:45 AM to 8:45 AM) by origin corridor

(Data source: Household Travel Survey, Chicago, Illinois, 1990)

the five corridors, the City of Chicago corridor is central to the network and the rest are radial. Extension of a radial transit line in a radial corridor was preferable because such an extension is more likely to have minimal or no impact on the other corridors; hence the effect of extension as predicted by different models in the unaffected corridors would be more visible. Consequently, the City of Chicago corridor was not an option from this point of view. Another reason for excluding the City of Chicago corridor from the line extension consideration can be explained by Figure 15 through 17. These figures show that the City of Chicago is very well served by all the transit services, especially by the CTA buses and trains, whereas the other corridors are mainly served by the radial Metra commuter trains. Because of the intensive transit service available in the City of Chicago, any change in some of the transit lines would produce any sizable difference in the transit cost component and hence in the ridership pattern in the region. Among the rest of the corridors, the West Corridor was chosen as the corridor with the transit line extension, which is called the Extension Corridor later on. The West corridor was chosen as the extension corridor because it has the highest volume of Suburb-to-CBD transit traffic, as can be seen in Figure 20. The West corridor, i.e. the extension corridor, is mainly served by Metra Lines no. 5 and 6 (Figure 8); both lines were extended radially as shown in Figure 21. These extensions took the transit lines into some zones in the west corridor that had heavy volumes of originating trips, i.e. origins that were not being directly served by any transit line such as zones 319, 320, 701, 703, 901 etc.

Transit line extensions affect access to transit and hence the generalized cost of travel by transit in some zones, which will be called the Affected Zones (Figure 18). A total of 16

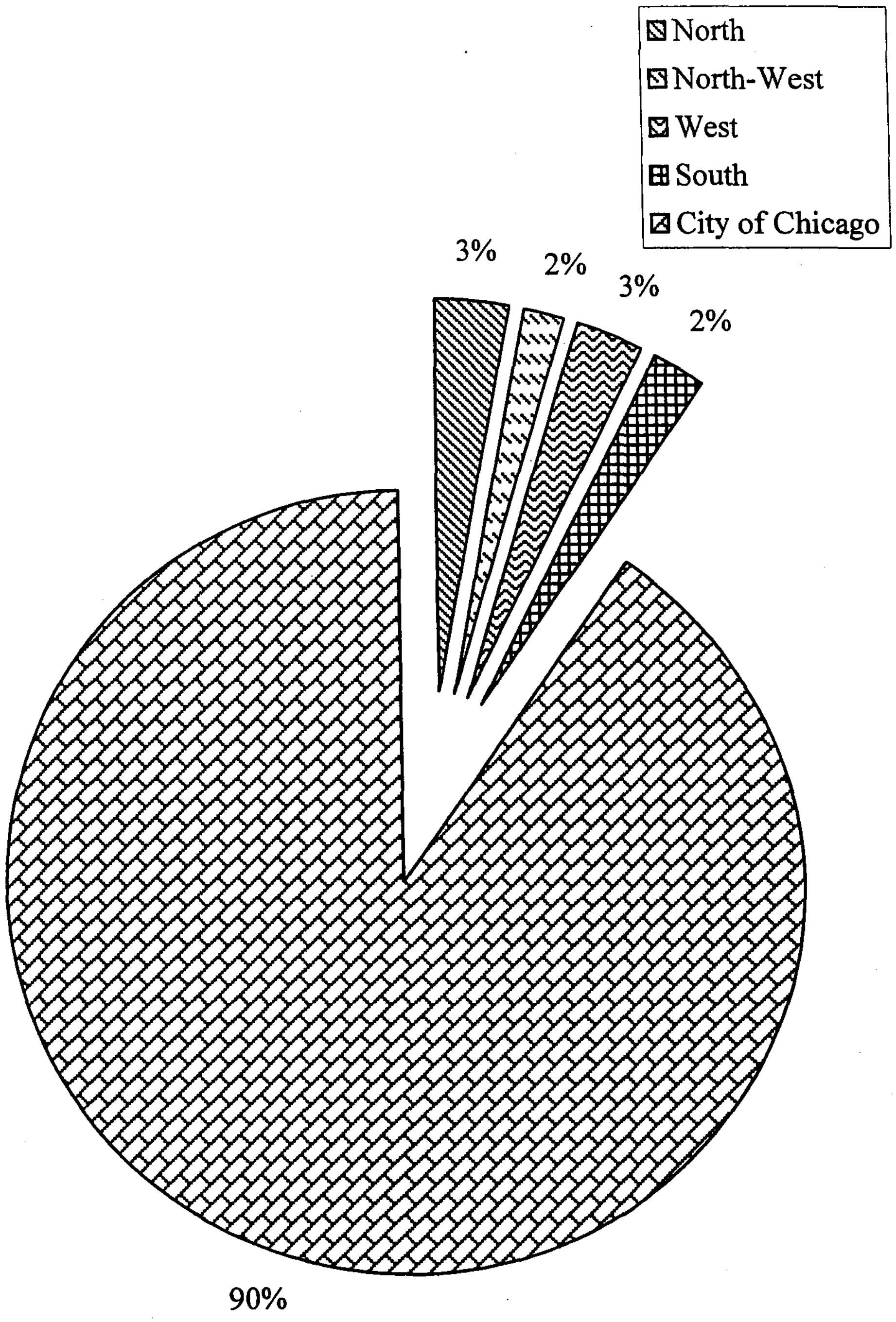


Figure 15: Distribution of CTA bus ridership originating in different corridors in the Chicago Region

(Data source: Household Travel Survey, Chicago, Illinois, 1990)

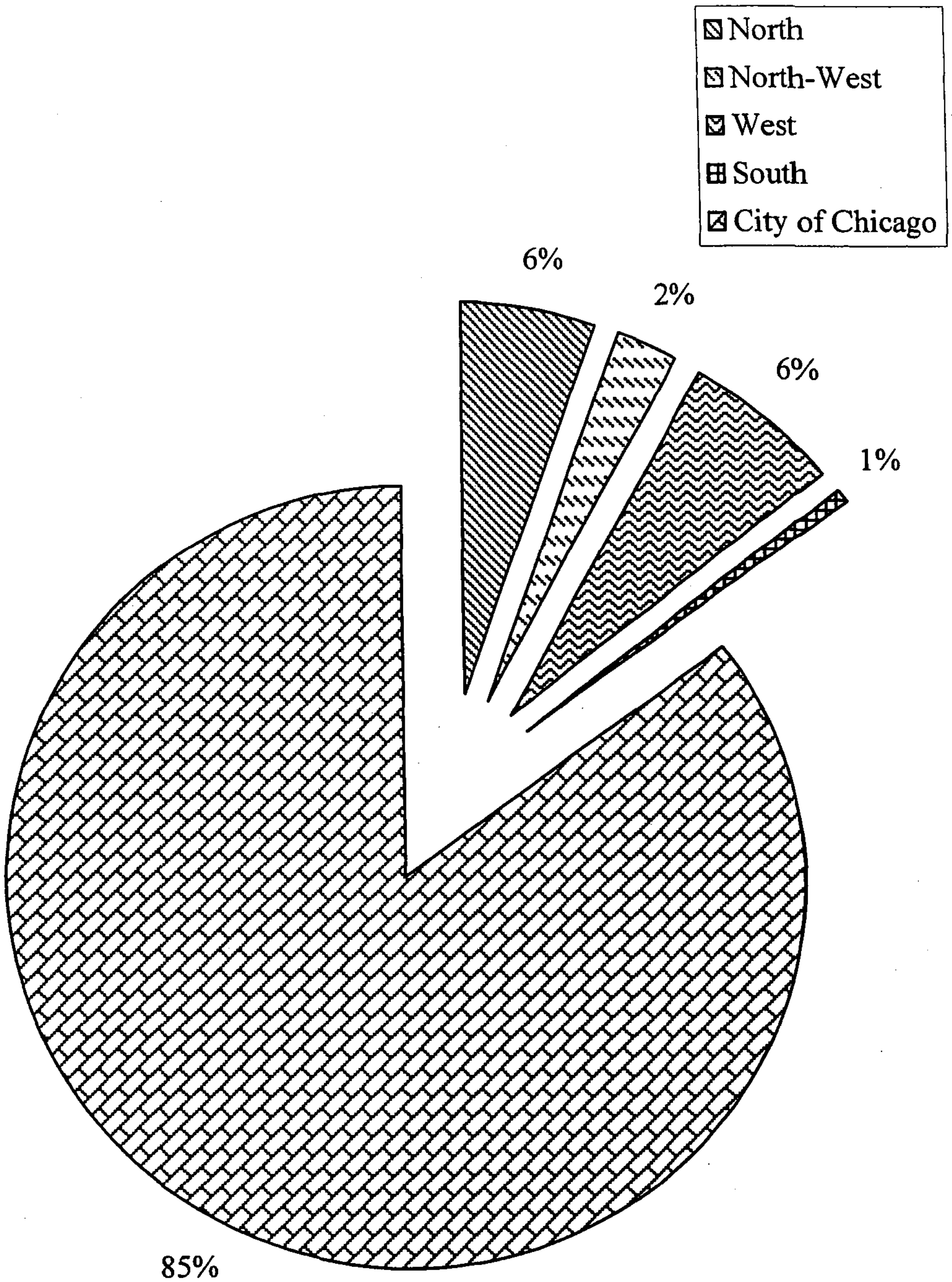


Figure 16: Distribution of CTA train ridership originating in different corridors in the Chicago Region

(Data source: Household Travel Survey, Chicago, Illinois, 1990)

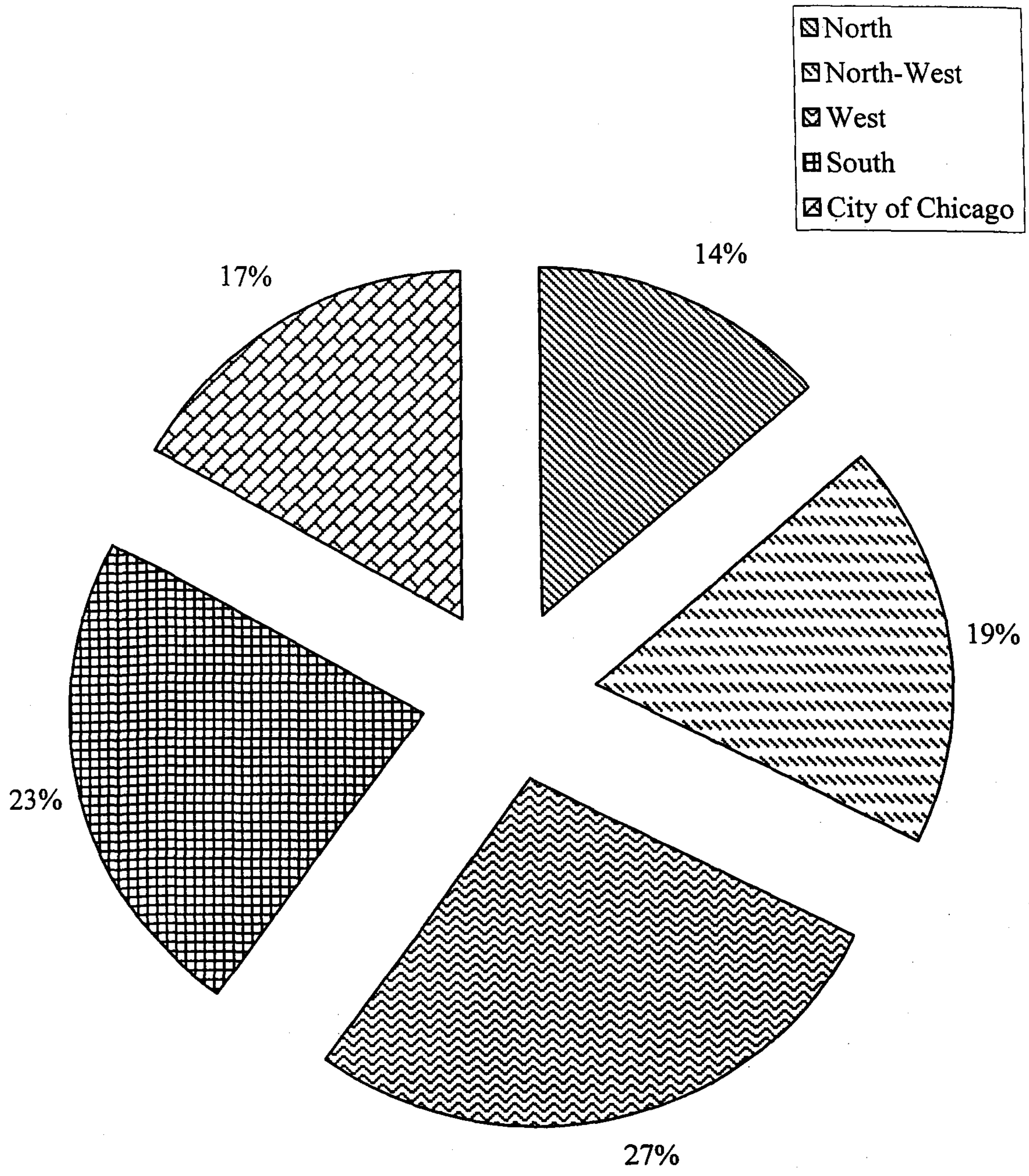


Figure 17: Distribution of Metra train ridership originating in different corridors in the Chicago Region

(Data source: Household Travel Survey, Chicago, Illinois, 1990)

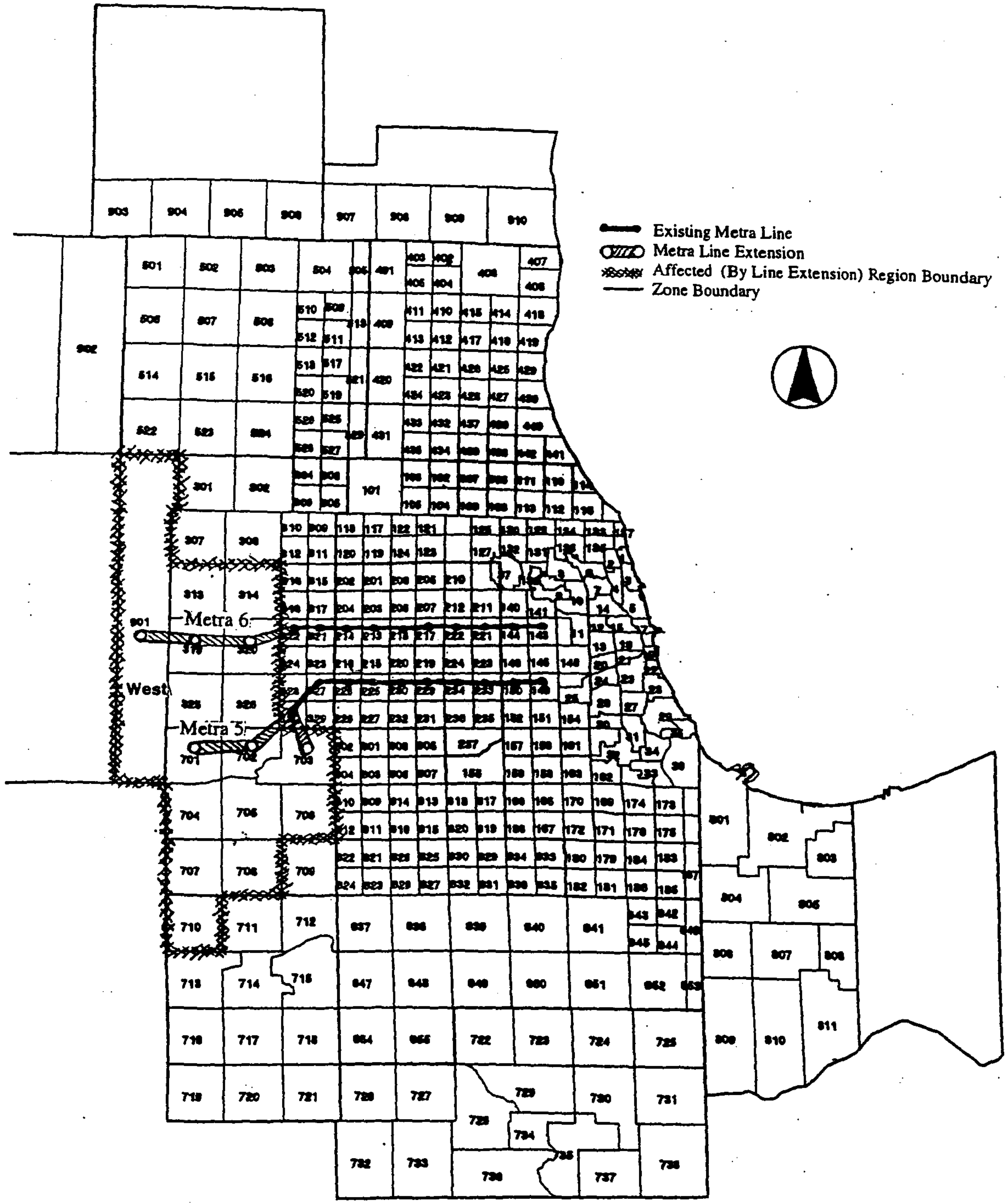


Figure 18: Extension of Metra Line No. 5 and 6 in the West Corridor and Affected Zones

affected zones were selected upon visual examination of the network. The affected zones include all zones along the extended portion of the lines and some other neighboring zones. Although the affected zones may well be beyond the boundaries of the extension corridor, thereby reorganizing the geographic structure of the corridors, such a corridor restructuring effect of the transit line extension was ignored and the affected zones were kept all within the extension corridor. This approach greatly simplifies the implementation of the transit line extension in terms of the changes in the transit data. This approach is also justifiable in the following sense. The current research focussed on the comparative behavior of the 1D and 2D Combined Models with regard to the line extension. Since the model implementations used here does not use transit assignment, corridor restructuring would be rather artificial and introduce ambiguity in model behavior.

As described in the previous section, because of high volume of CBD bound transit traffic in the morning peak period, the CBD was selected as the target destination for the comparative study of model behavior. As a result, and also to keep line extension implementation simple, transit cost components between the 16 affected zones and the four CBD zones i.e. a total of 64 zone pairs were altered. A very simple approach was adopted for making these necessary changes.

For each transit line that was extended, all the zones along that line, starting from the non-CBD end of the existing line and going towards the CBD-end, were identified. Then, all the transit cost components from these zones to all of the four CBD zones were compiled. Finally, for each CBD zone, these IVT, OVT and fare values were extrapolated to obtain the IVT, OVT and fare respectively to that CBD zone, from the zones on the extension part of the line under consideration. The extrapolation was carried out in order to maintain the apparent trend in the variation of the to-CBD transit IVT and fare along the existing portion of a line, to the extension portion as well. In the case of OVT, it was arbitrarily assumed that the to-CBD OVT for the zones on the extension segment would be the same as that of the zone at the end of the existing portion of the corresponding line. The end result of this simple empirical procedure is shown in Figures 19 through 24.

For the zones that are affected by line extension but are not along the extension segments, a different simple scheme was employed. It was assumed that the travelers in those zones, who want to make a transit trip, would drive to the nearby zone that is directly on a transit line, subsequently called the Boarding Zone, and take transit from there. As a result, the IVT and fare for these zones are the same as those of the corresponding boarding zones and the OVT will be the OVT of the corresponding boarding zones plus the driving time i.e. the auto access time to the boarding zone. The boarding zones were selected by visual examination of the network map after the transit extension. In the vicinity of the affected zones there are mainly two sizes of zones: 36 square miles and 9 square miles. The auto access times between two 36 square miles zones, two 9 square miles zones, and a 36 square miles zone and a 9 square miles zone were assumed to be 9 minutes, 5 minutes and 7 minutes respectively. These values were reasonable considering the freeway access and higher arterial speed limit to the travelers in those suburban

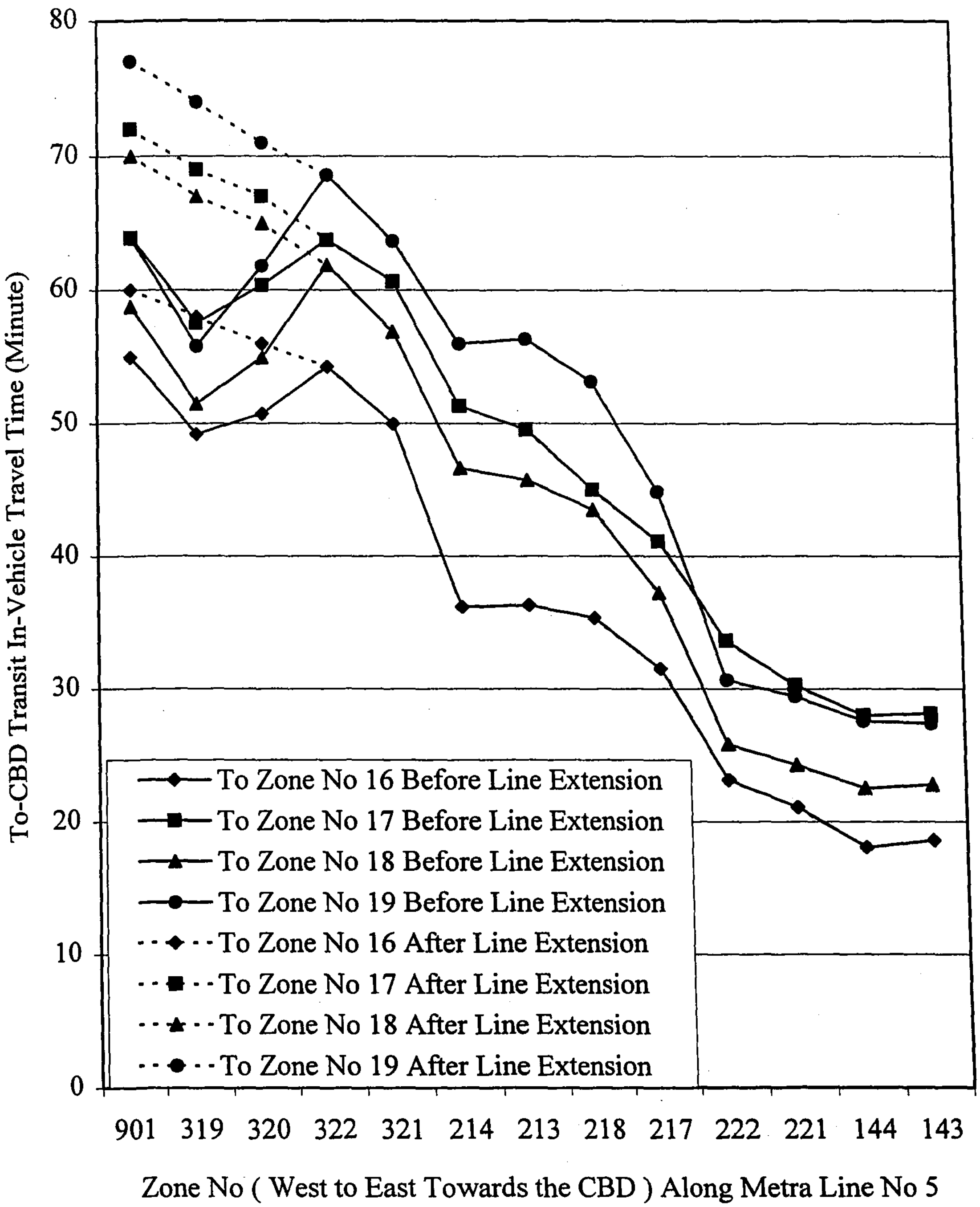


Figure 19: Variation of to-CBD transit IVT along Metra line no. 5

(On the average an increase of 10.23 minutes)

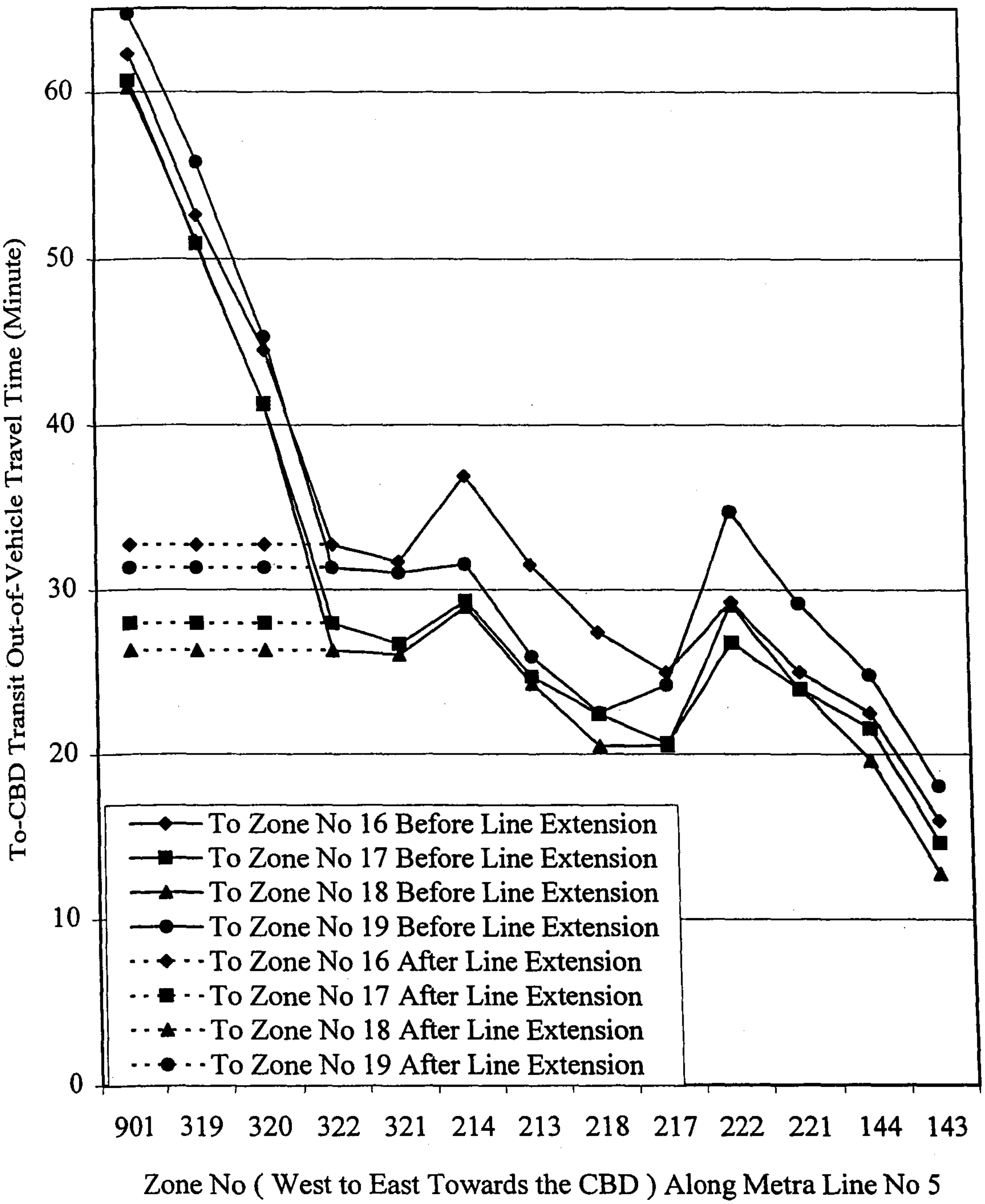


Figure 20: Variation of to-CBD transit OVT along Metra line no. 5

(On the average a decrease of 23.07 minutes)

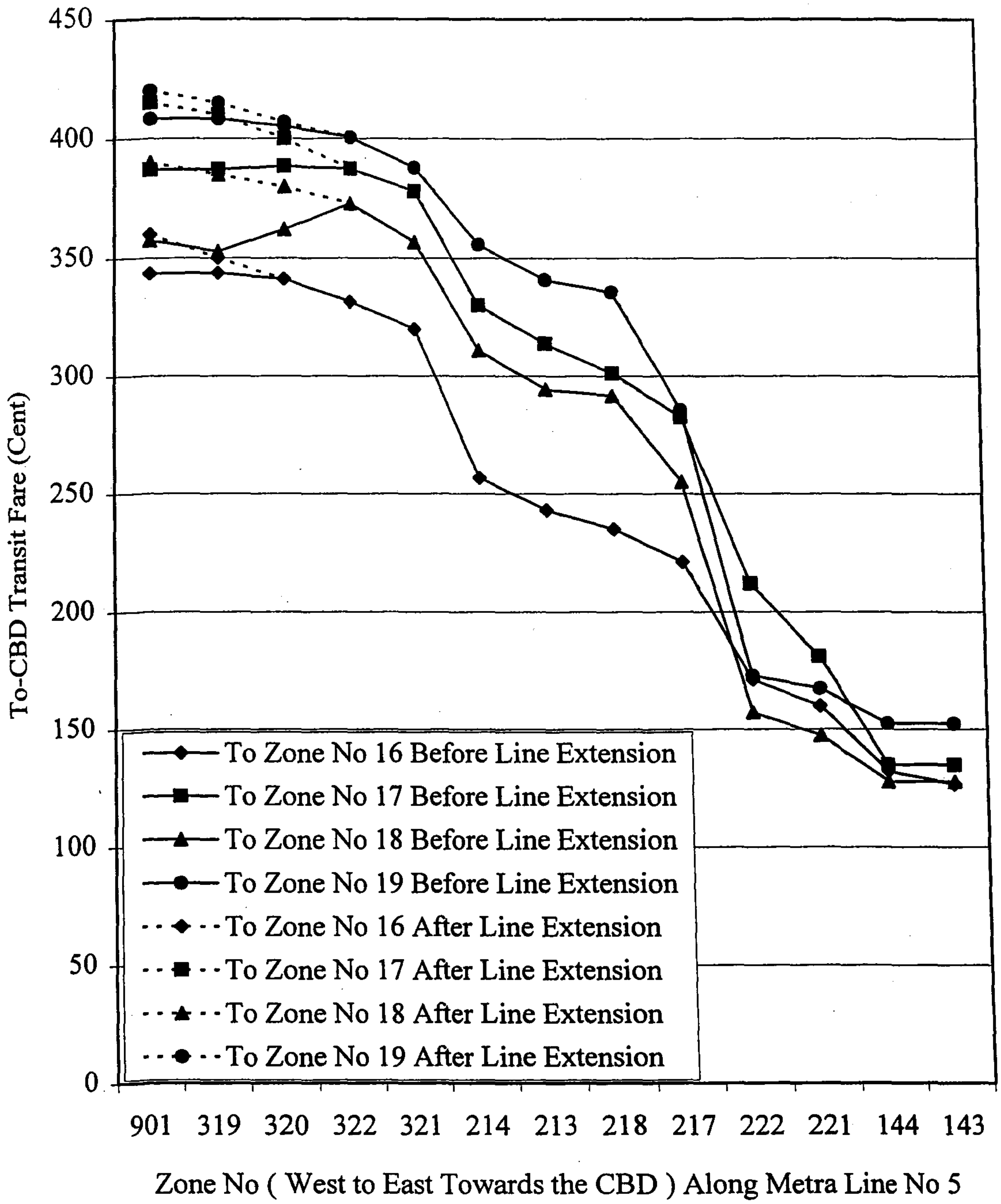


Figure 21: Variation of to-CBD transit fare along Metra line no. 5

(On the average an increase of 15.8 cents)

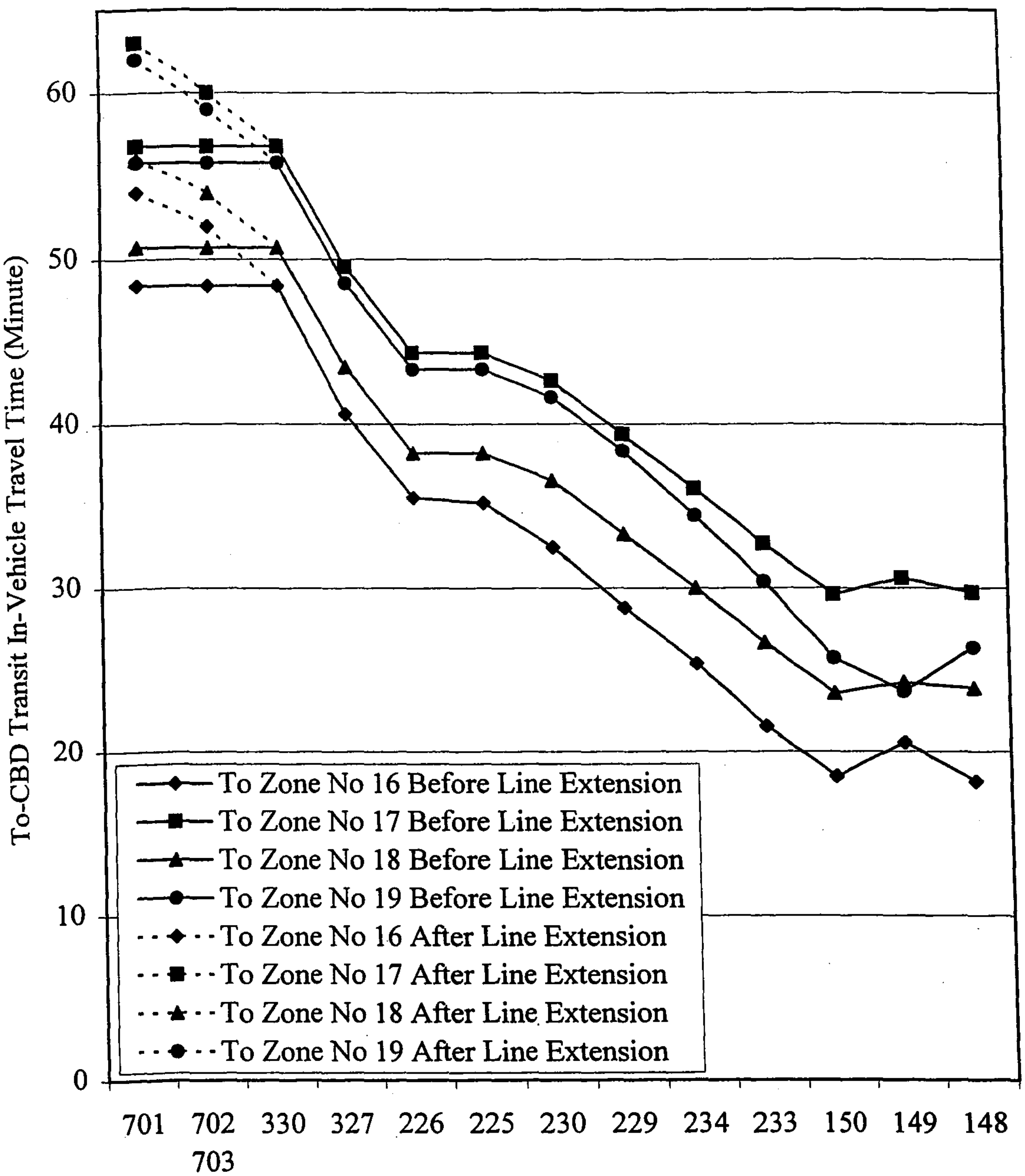


Figure 22: Variation of to-CBD transit IVT along Metra line no. 6

(On the average an increase of 4.16 minutes)

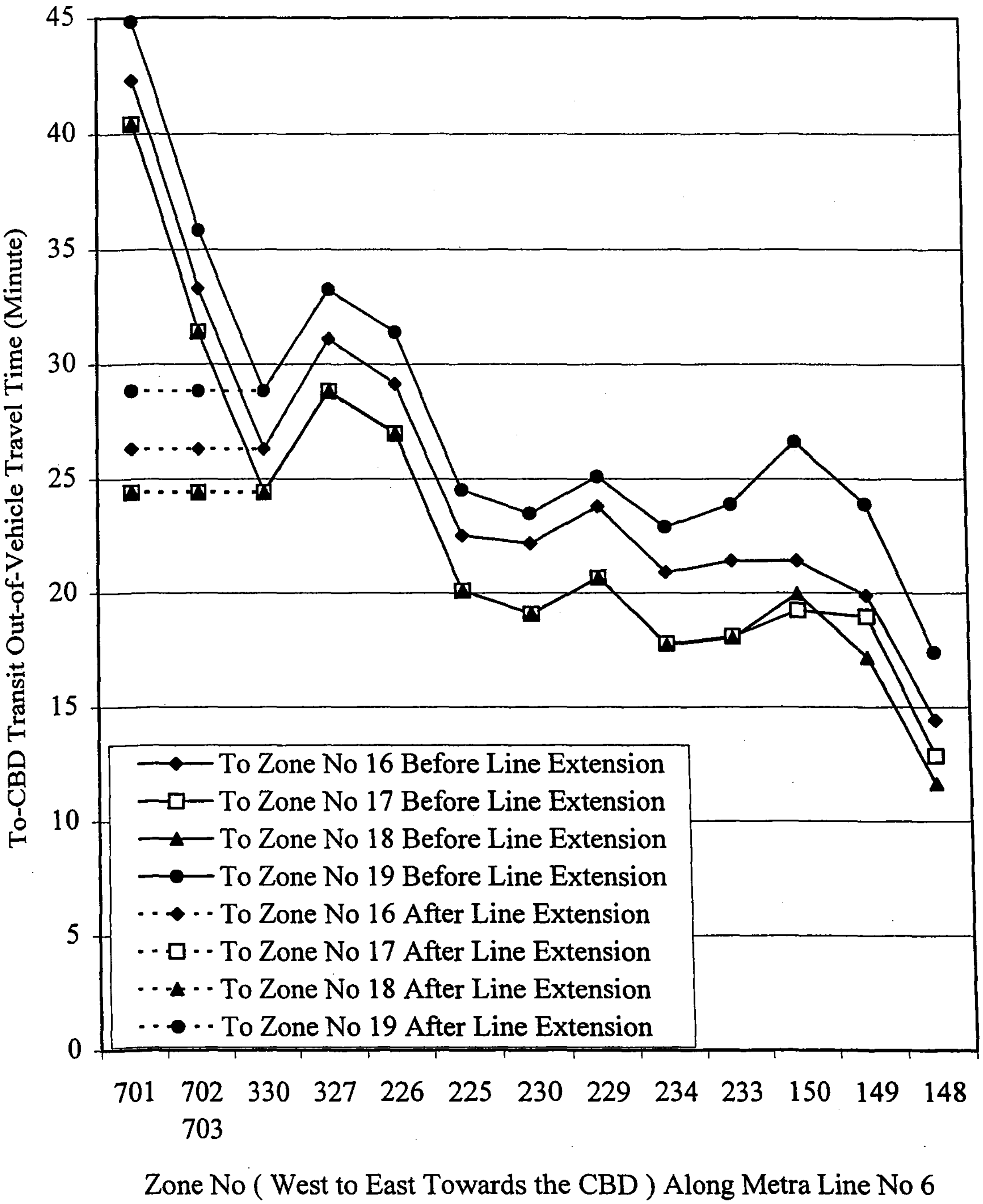


Figure 23: Variation of to-CBD transit OVT along Metra line no. 6

(On the average a decrease of 9.25 minutes)

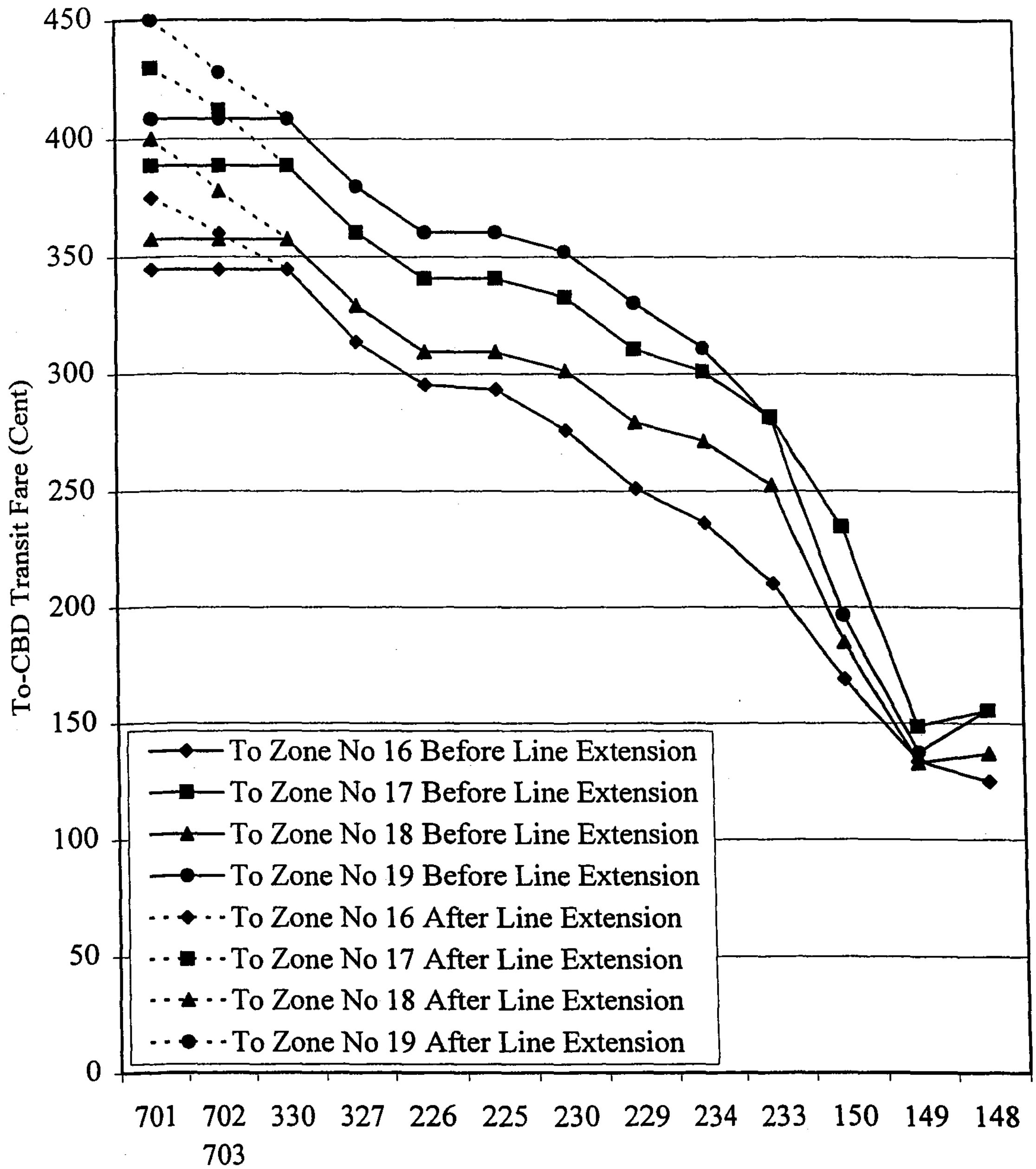


Figure 24: Variation of to-CBD transit fare along Metra line no. 6

(On the average an increase of 26.2 cents)

zones. However, if these changes increased the generalized travel cost of transit between a zone pair, the cost component between that zone pair was not changed. The changes made in the cost components of the affected zones not along any transit line are presented in Figures 25 through 27.

Figure 28 summarizes the changes in the to-CBD generalized transit travel cost from the affected zones resulting from the proposed hypothetical transit line extension. The generalized cost of travel was computed as follows:

$$\text{Generalized Cost} = \sum \text{Cost Component} \times \text{Respective Cost Component Parameter} \quad (4.1)$$

Finally, a separate set of transit data, i.e. the Extended Transit Data, was prepared by incorporating the changes in IVT, OVT and fare into the Existing Transit Data.

4.3 Effect of Origin-Destination Constraints in the Combined Model on Transit Line Extensions

The trip matrix for transit predicted by the Combined Model was used to calculate the total number of transit trips originating in each corridor and going to the CBD. As described in the previous section, 16 zones at the west-end of the West corridor underwent a decrease in to-CBD transit travel cost due to the proposed transit line extensions. Accordingly, the relative attractiveness of transit in the West corridor was increased compared to auto so far as the CBD bound traffic was concerned. However,

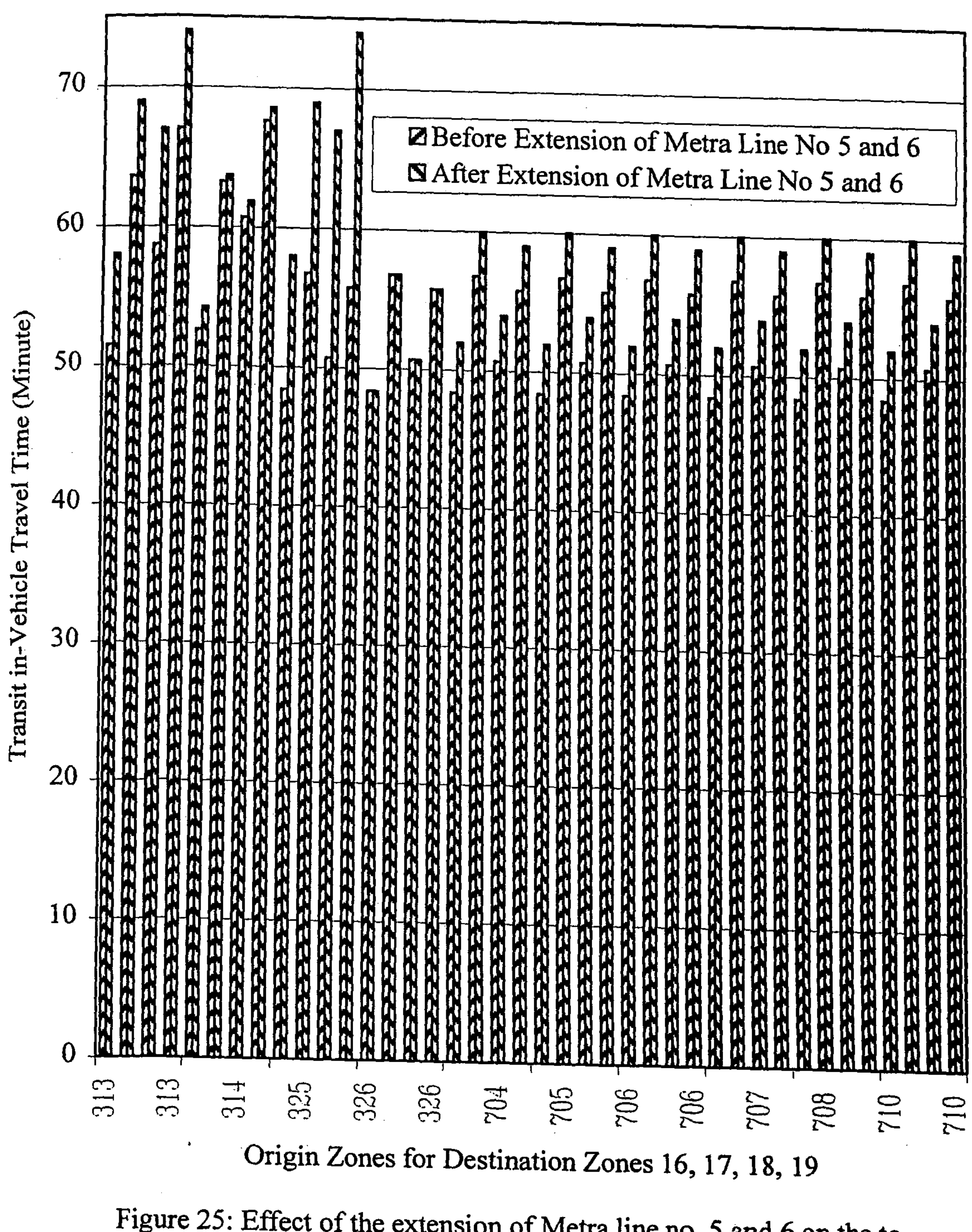


Figure 25: Effect of the extension of Metra line no. 5 and 6 on the to-CBD transit IVT of nearby zones, which are not along the lines

(On the average an increase of 4.19 minutes)

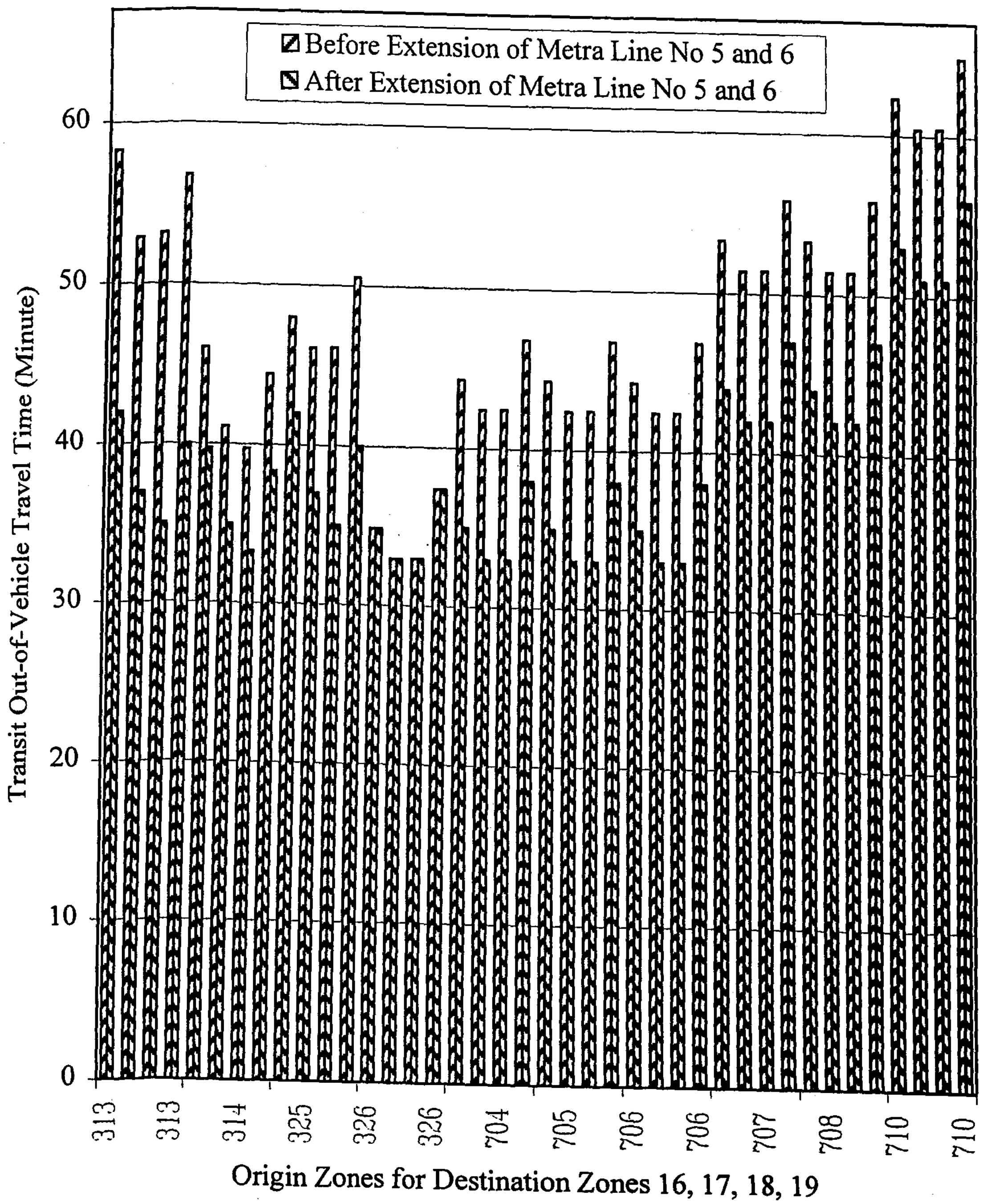


Figure 26: Effect of the extension of Metra line no. 5 and 6 on the to-CBD transit OVT of nearby zones, which are not along the lines

(On the average a decrease of 8.77 minutes)

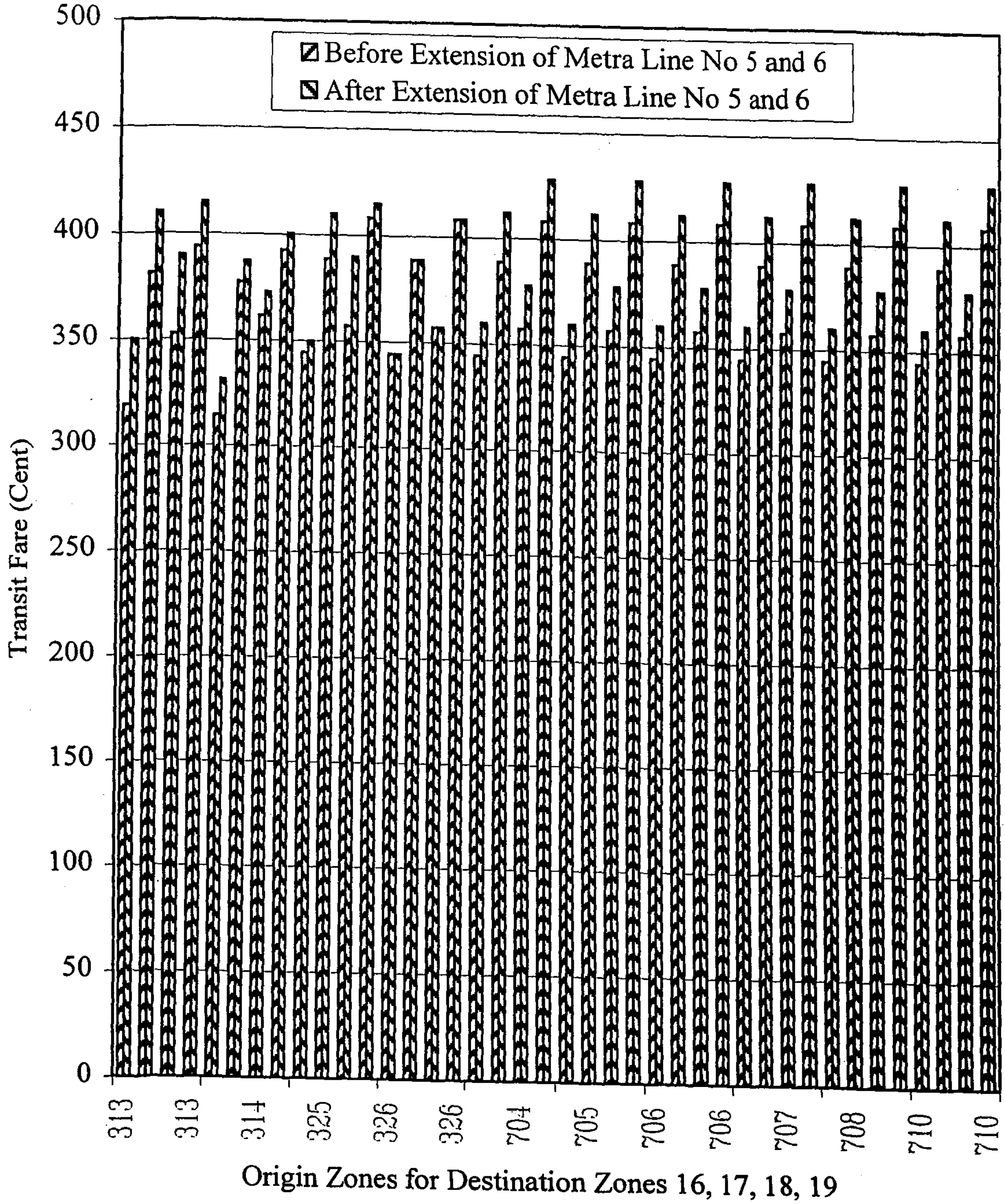


Figure 27: Effect of the extension of Metra line no. 5 and 6 on the to-CBD transit fare of nearby zones, which are not along the lines

(On the average an increase of 17.65 cents)

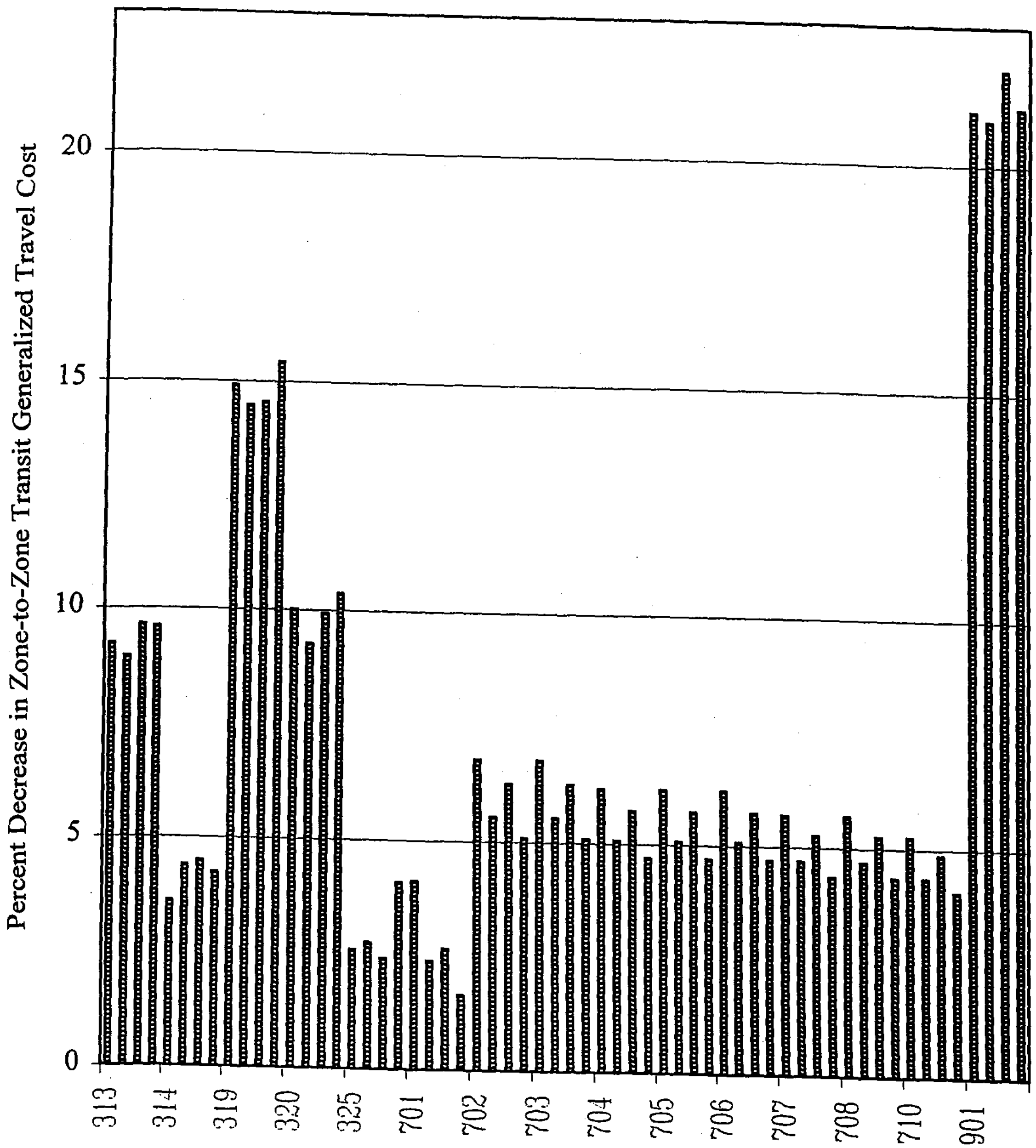


Figure 28: Reduction in zone-to-zone generalized cost of travel by transit in the Affected Zone-pairs after transit line extension

(Average reduction is 7.21%; Transit IVT, OVT and fare cost parameters are 0.034, 0.129 and 0.0169 respectively) this extension of transit service in the West corridor did not affect the transit cost components in the other corridors. As a result, the expected response of the model to such an extension would be that only travelers in the West corridor would be affected by the reduced transit cost components. Hence it would be expected that the model solution would show increased to-CBD transit ridership in the West corridor while ridership in the other corridors would be unaffected.

However, if the destination constraints are applied to the model formulation, as in the case of the 2D Combined Model, the model prediction is not in accord with this logical expectation. While solving the 2D Combined Model, in order to allocate a prefixed number of trips to the CBD destinations, any increase in to-CBD traffic from the West corridor must be offset by a decrease in CBD bound transit trips from the other corridors, where travel costs by transit remains unchanged. Since application of the destination constraints gives rise to this undesirable behavior of the 2D Combined Model, the expected behavior can be obtained by ignoring such constraints, i.e. by using the 1D Combined Model.

As discussed in the previous section, the difference between the 2D Existing and 2D Extended Transit Solutions should demonstrate the 2D Combined Model response to a transit line extension. On the other hand, the difference between the 1D Existing and Extended Transit Solution reveals how the 1D Combined Model responds to the effect of transit line extensions. Table V enumerates the to-CBD transit ridership for different corridors as predicted by the 1D and 2D Combined Models before and after the

TABLE V

CBD BOUND TRANSIT RIDERSHIP PREDICTED BY DIFFERENT COMBINED MODELS

Corridor	2D Existing Transit Solution	2D Extended Transit Solution	1D Existing Transit Solution	1D Extended Transit Solution
North	4142.52	4141.69	4143.25	4143.22
North-West	3843.08	3841.80	3843.13	3843.14
West	8563.76	8579.03	8562.48	8580.24
South	8044.33	8043.14	8047.98	8048.06
City of Chicago	70819.62	70811.75	70822.18	70822.21

hypothetical transit line extension. The results show the behavioral difference of the 1D and 2D Combined Model. Figure 29 compares the changes in transit ridership predicted by the 1D and 2D Combined Models in response to transit service extension. For the 2D Combined Model, Figure 29 shows that an increase in transit ridership in the extension corridor is indeed accompanied by a decrease in transit ridership in the other corridors. In contrast, Figure 29 reveals that the 1D Combined Model only predicts increased transit ridership in the West corridor, where the extension took place, and leaves the ridership in the other corridors unchanged within limits of computational accuracy. However, Figure 29 and Table V reveal that the degree of change is rather small in case of both the

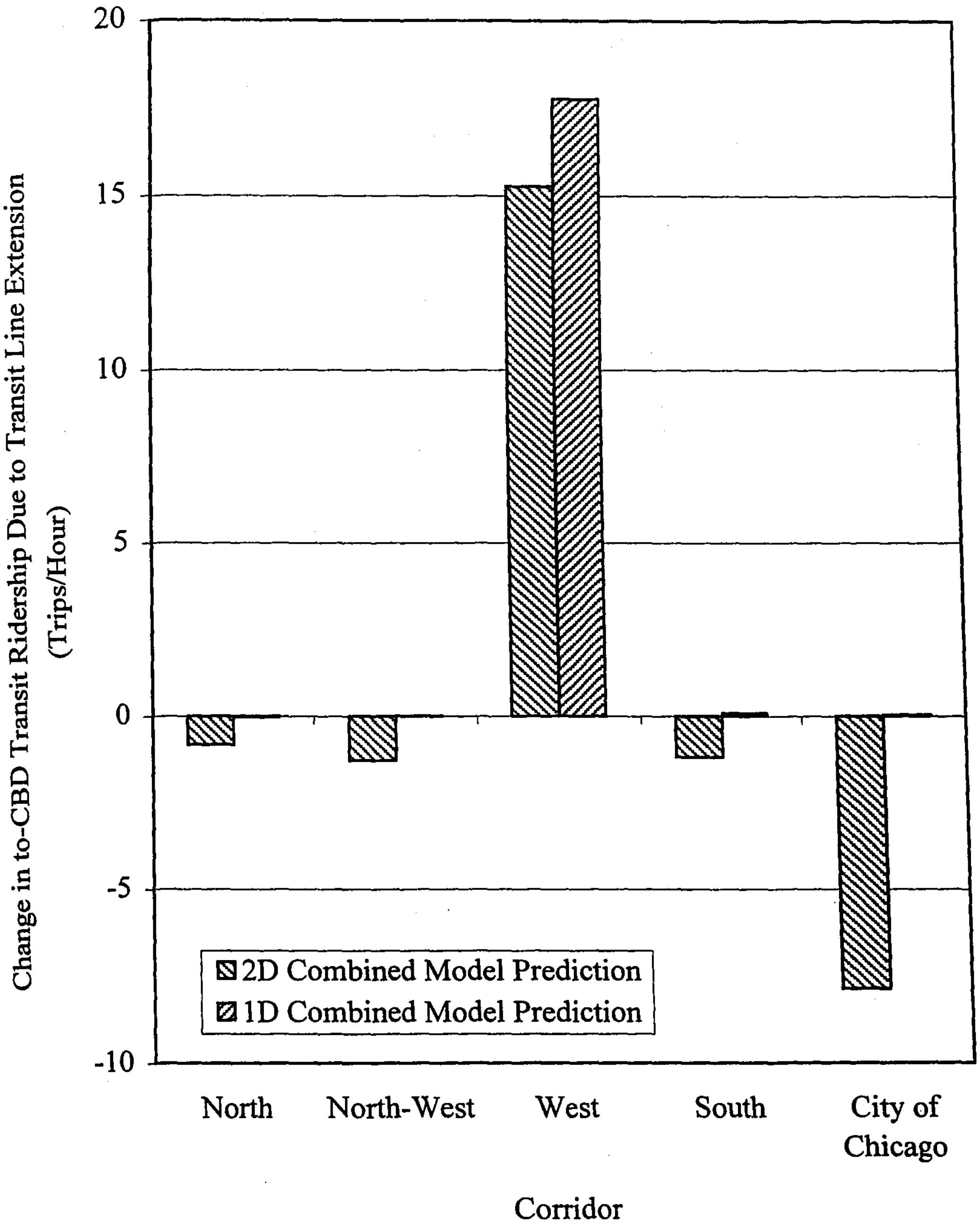
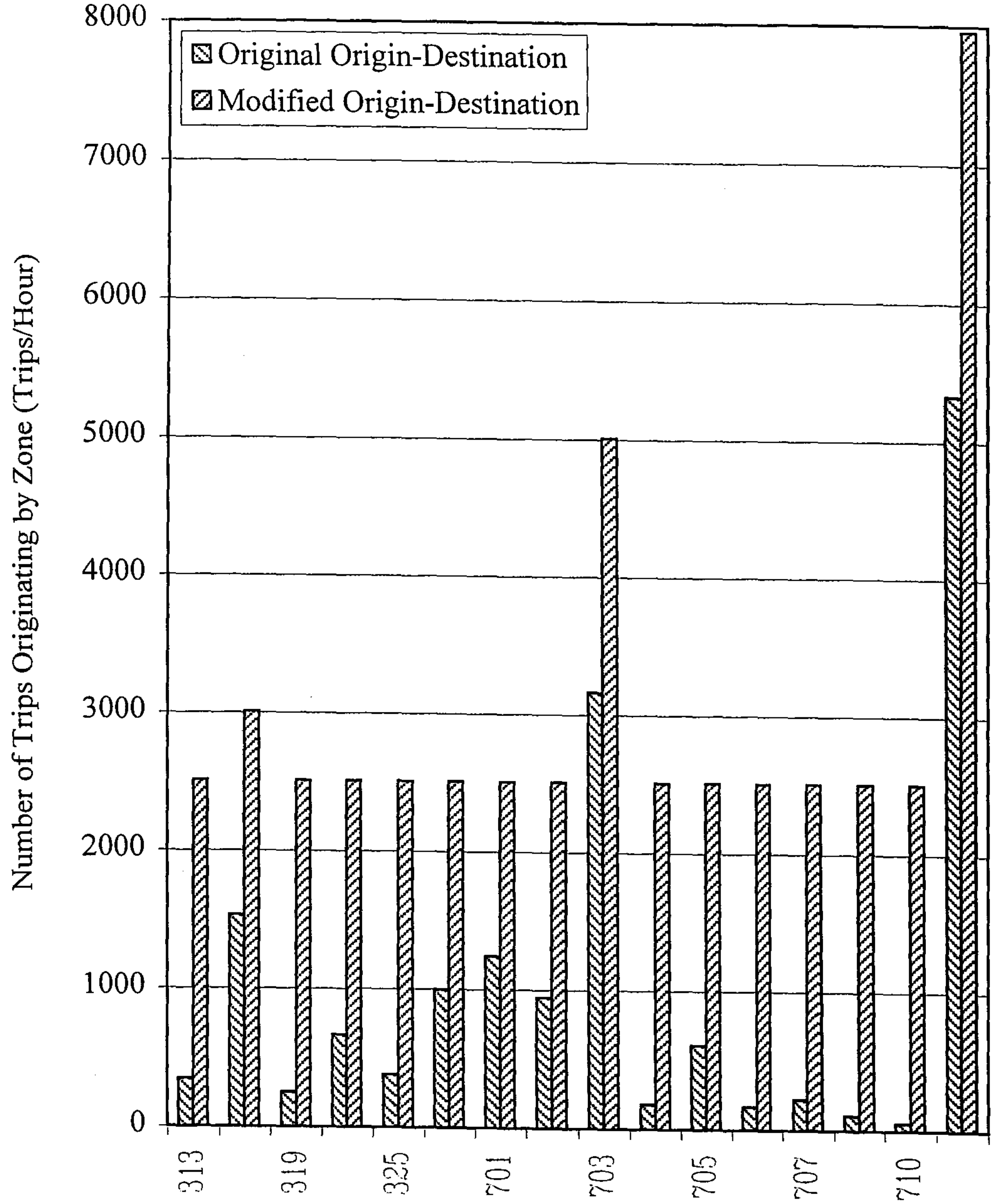


Figure 29: Comparison of to-CBD transit ridership change in different corridors due to transit line extension as predicted by 2D and 1D Combined Models using original Origins-Destinations

(Transit line was extended in the West corridor)

models. Especially, the transit ridership reduction in corridors other than the Extension Corridor is modest as predicted by the 2D Combined Model. Two major reasons for the small values of changes are as follows. First, the proposed extension resulted in a small reduction in generalized transit travel cost in the affected zones (Figure 28). Second, only a few of the affected zones had a substantial number of trips originating in those zones; therefore not many travelers were able to take advantage of the transit travel cost reduction produced by the line extension. Adjustment of the transit cost components to reflect the effect of transit line extension was done empirically and was justified to a considerable extent. In order to magnify the effect of transit line extension it was decided that the trip origins of the affected zones should be arbitrarily increased, on the assumption that the line extensions were coordinated with suburbanization of the zones.

Figure 30 shows the changes made in the number of trips originating in the affected zones. It is necessary that the sum of the destinations (Ds) and the sum of the origins (Os) be equal. In order to implement this equality between the total origins and the total destinations, number of trips entering different zones had to be adjusted too, which was done as follows. The total increase in the origins was calculated. The average increase per zone was then computed. Destinations for each zone were increased by this average amount. The origins and destinations before the modification are called the Original Origins-Destinations and those after the modification is referred to as the Modified Origins-Destinations. Figure 31 shows that these modifications of the origins and destinations (Os and Ds) indeed magnified the effect of transit line extension.



Zones Affected By Transit Line Extension (313, 314, 319, 320, 325, 326, 701, 702, 703, 704, 705, 706, 707, 708, 710, 901)

Figure 30: Number of Origins in the zones affected by transit line extension

(Total Origins changed by 2.34% from 1383589 to 1415991 trips/hour; on the average an increase of 200 percent in the Affected Zones; total increase is 32,402 trips/hour)

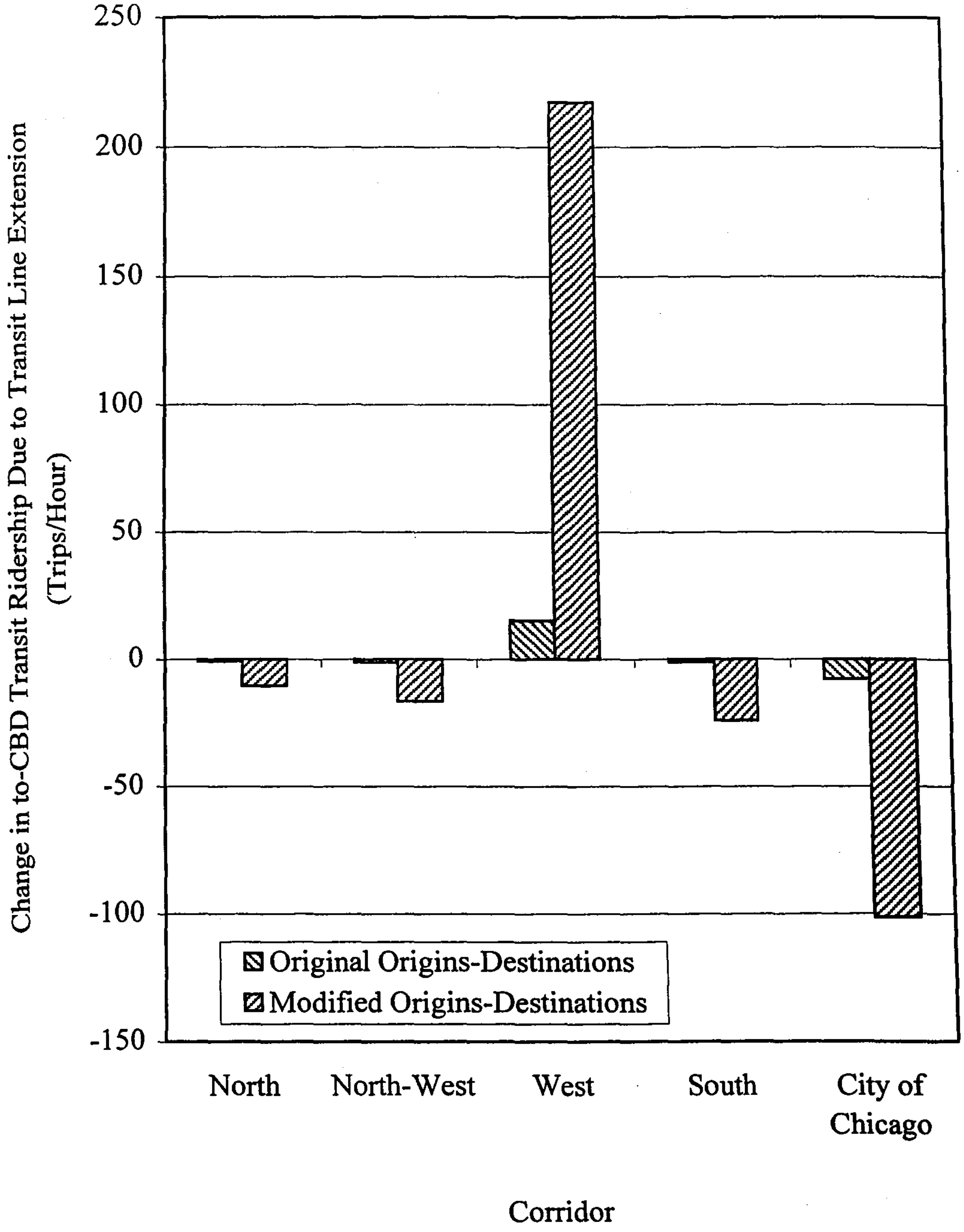


Figure 31: Comparison of to-CBD transit ridership change by corridors due to transit line extensions as predicted by the 2D Combined Model using original and modified Origins-Destinations

(Transit line extended in the West corridor)

Figure 32 demonstrates once again, at a larger scale with the Modified Origins-Destinations, that although both the 1D and 2D Combined Models show an increase in transit ridership in the extension corridor, their predictions are markedly different for the other corridors. While the 2D Combined Model predicts an undesirable decrease in transit ridership in the other corridors, the 1D Combined Model does not show any such change.

4.4 Effect of Highway Capacity on Model Behavior with Respect to Transit Line Extensions

Use of highways has a direct interaction with the use of transit. Among other factors, the relative attractiveness of transit over auto depends on the level of congestion on the highway network. The lower the highway capacity the greater is the highway congestion. As a result, with a decrease in highway capacity, auto IVT increases resulting in an increased cost of travel. Consequently transit is more attractive to cost-sensitive travelers. If some transit lines are extended, more travelers will take advantage of the reduced cost of travel by transit due to the extension.

With the higher ridership in the extension corridor, the unexpected decrease in transit ridership in the other corridors shown by the 2D Combined Model is higher too. Figures 33 and 34 show that a hypothetical decrease in highway capacity intensifies the anomalous behavior of the 2D Combined Model. Figure 35 and 36 show that the 1D

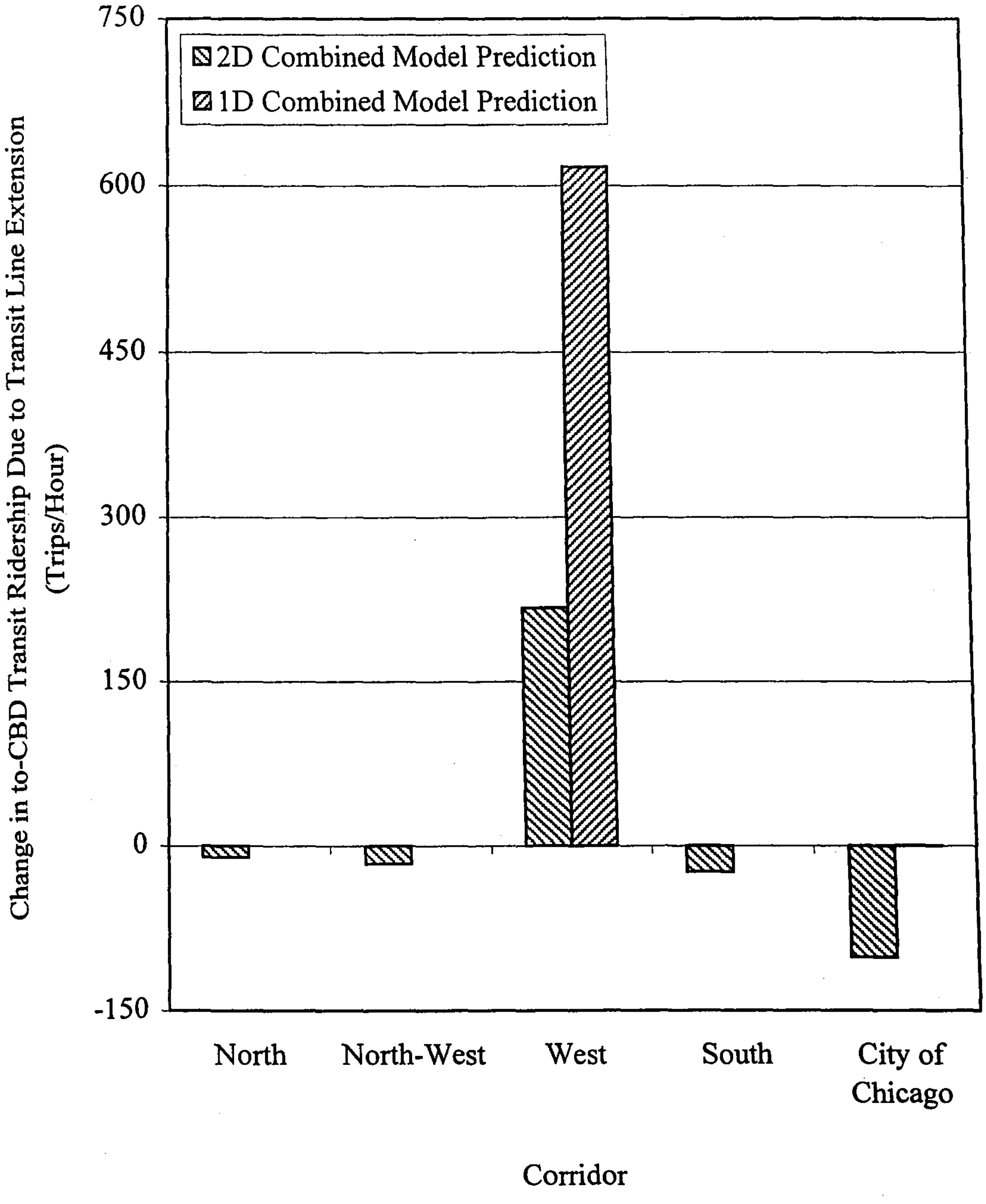


Figure 32: To-CBD transit ridership change by corridors due to transit line extension as predicted by the 1D and 2D Combined Model using modified Origins-Destinations

(Transit line extended in the West corridor)

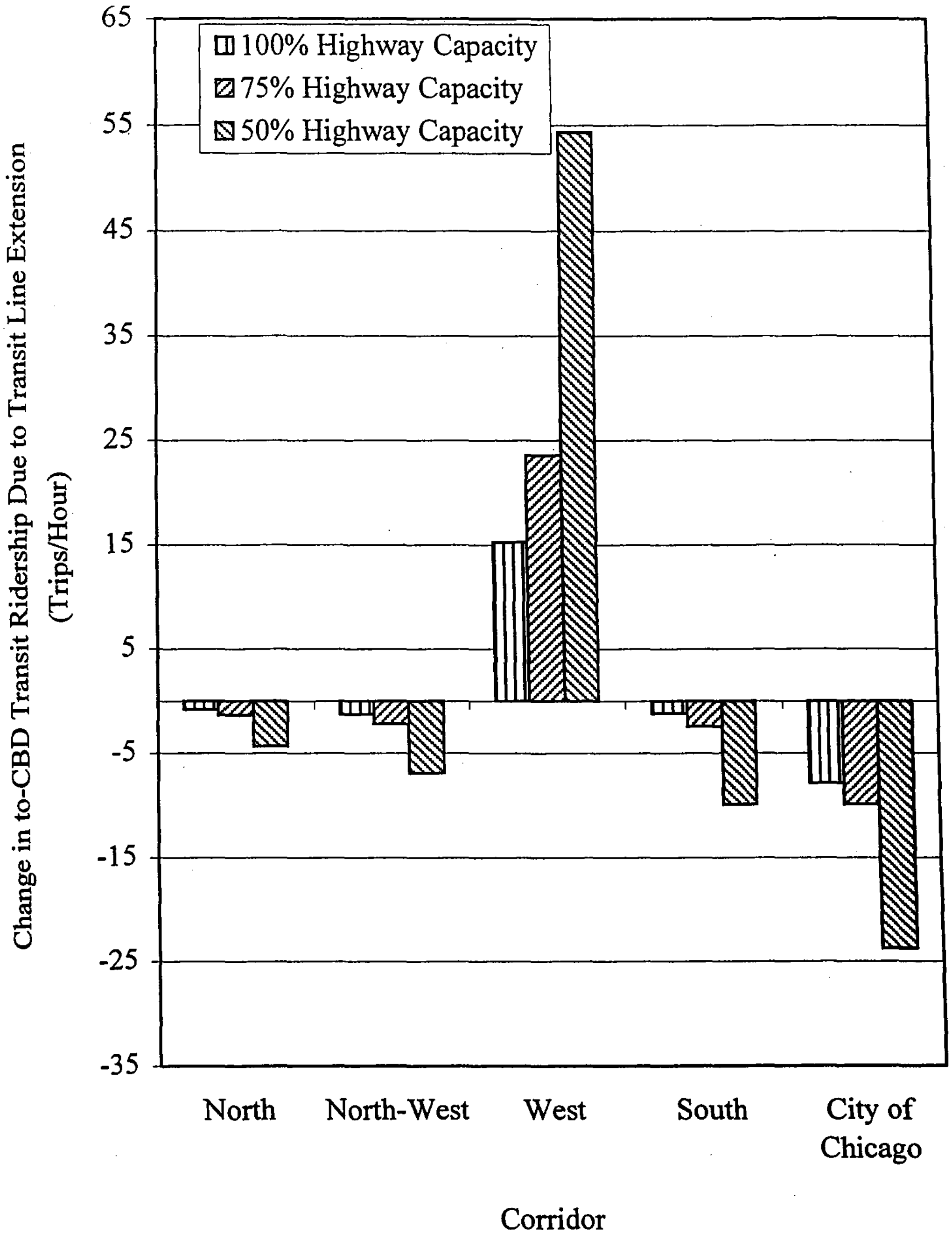


Figure 33: To-CBD transit ridership change by corridors due to transit line extension as predicted by the 2D Combined Model w.r.t. highway capacity using original Origins-Destinations

(Transit line extended in the West corridor)

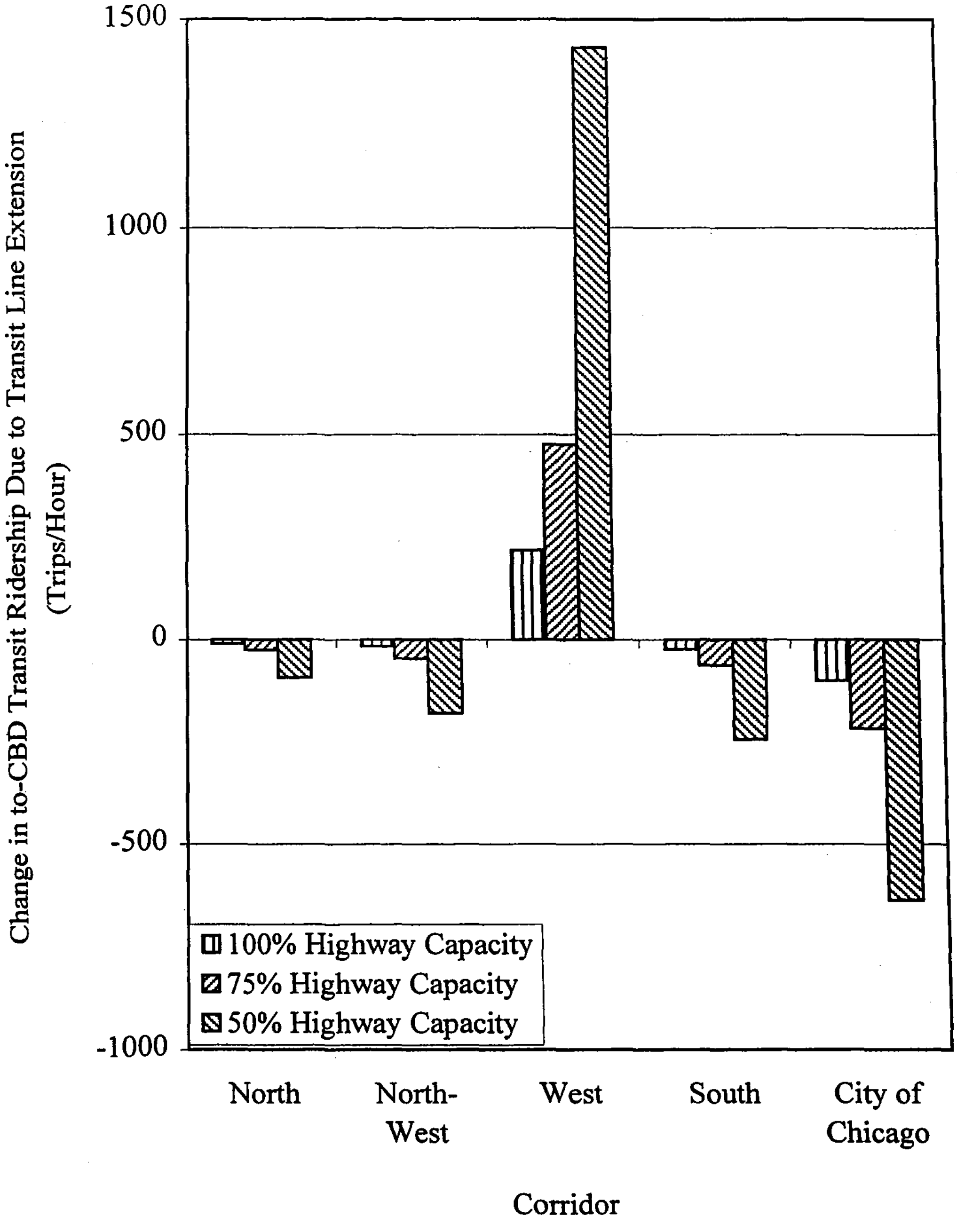


Figure 34: To-CBD transit ridership change by corridors due to transit line extension as predicted by the 2D Combined Model w.r.t. highway capacity using modified Origins-Destinations

(Transit line was extended in the West corridor)

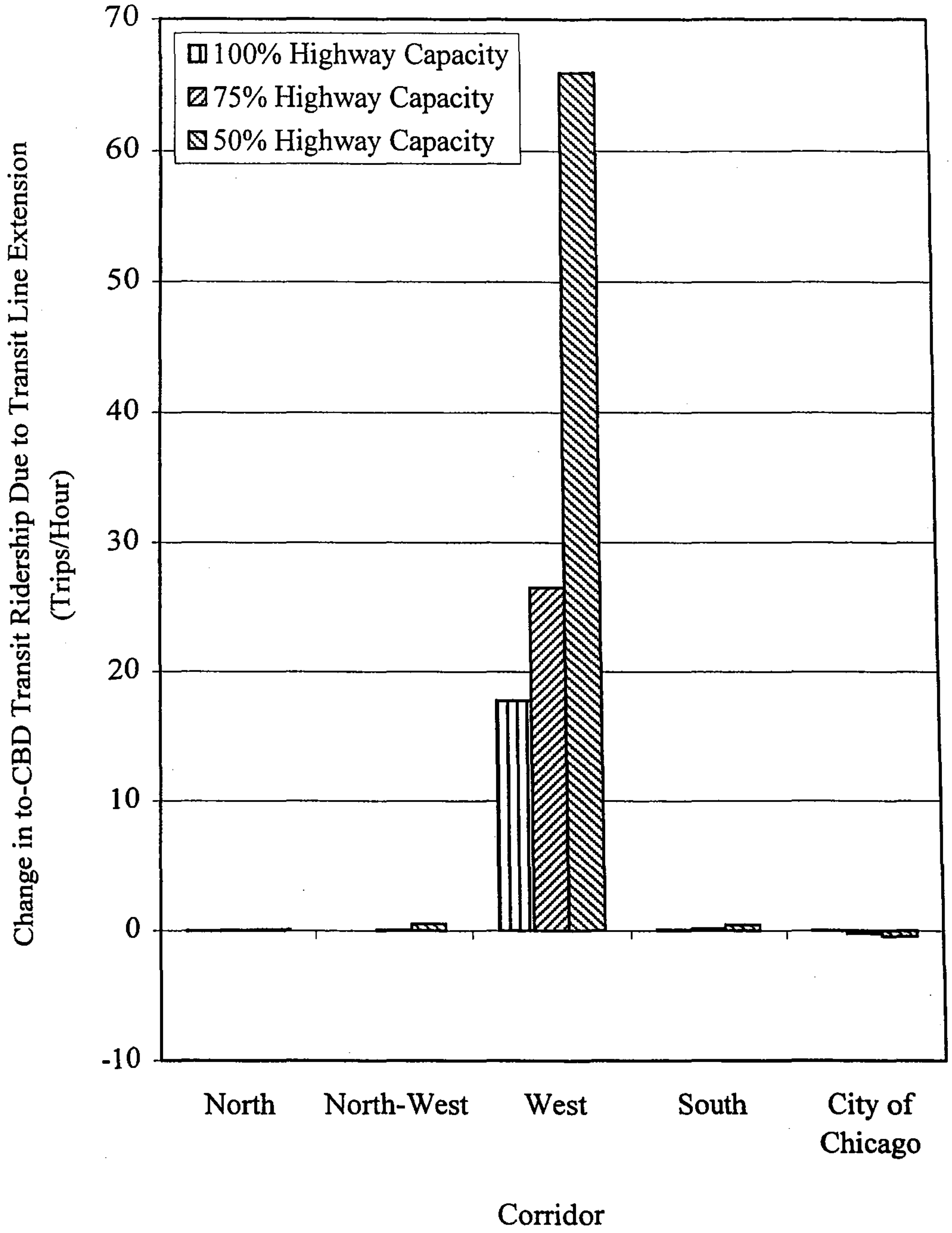


Figure 35: To-CBD transit ridership change by corridors due to transit line extension as predicted by the 1D Combined Model w.r.t. highway capacity using original Origins-Destinations

(Transit Line was Extended in the west Corridor)

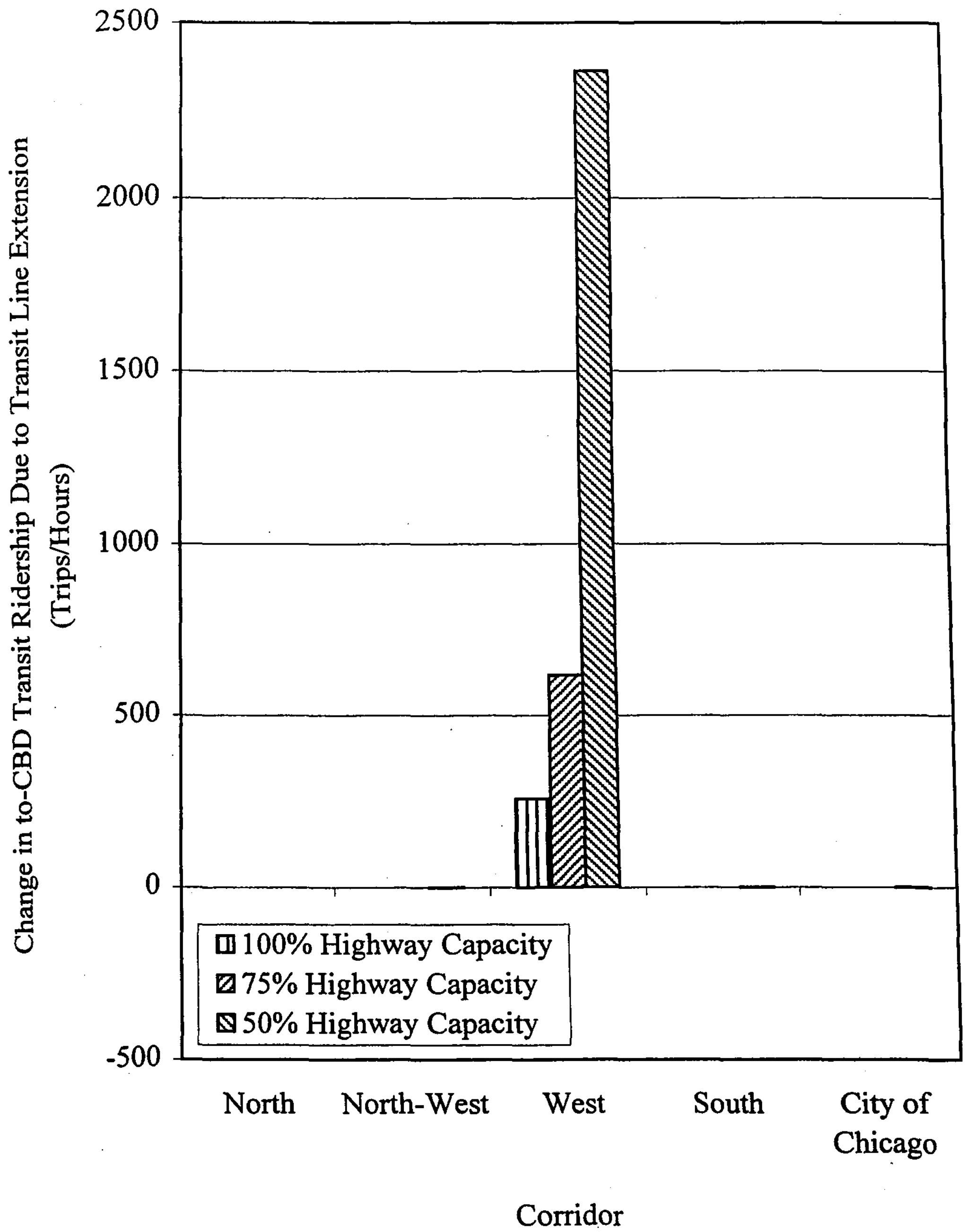


Figure 36: To-CBD transit ridership change by corridors due to transit line extension as predicted by the 1D Combined Model w.r.t. highway capacity using modified Origins-Destinations

(Transit Line was Extended in the west Corridor)

Combined Model makes consistent predictions by leaving the transit ridership in the corridors other than the extension corridor unchanged. It can be noted that the 1D and 2D Combined Models differ significantly in predicting the increase in transit ridership increase in the extension corridor. The 1D Combined Model prediction is considerably higher than that of the 2D Combined Model. Figure 37 shows that the difference in magnitude of the West corridor CBD bound transit ridership as predicted by the 1D and 2D Combined Models grows as highway capacity decreases, i.e. as highways become more congested.

4.5 Effect of Transit Line Extensions on Travel by Auto

Because of the interaction between the origin-destination and mode choice, changes in the To-CBD transit ridership due to line extension is likely to result in a restructuring of the mode choice. Figures 38 and 39 show that after the transit line extension, although there is no change in the To-CBD auto traffic when the 1D Combined Model is used, volume of such traffic reduces all over the network in case of the 2D Combined Model. It can be noted that the 2D Combined Model predicts a considerable reduction in the CBD bound auto traffic for some corridors. The changes intensify as the network capacity decreases.

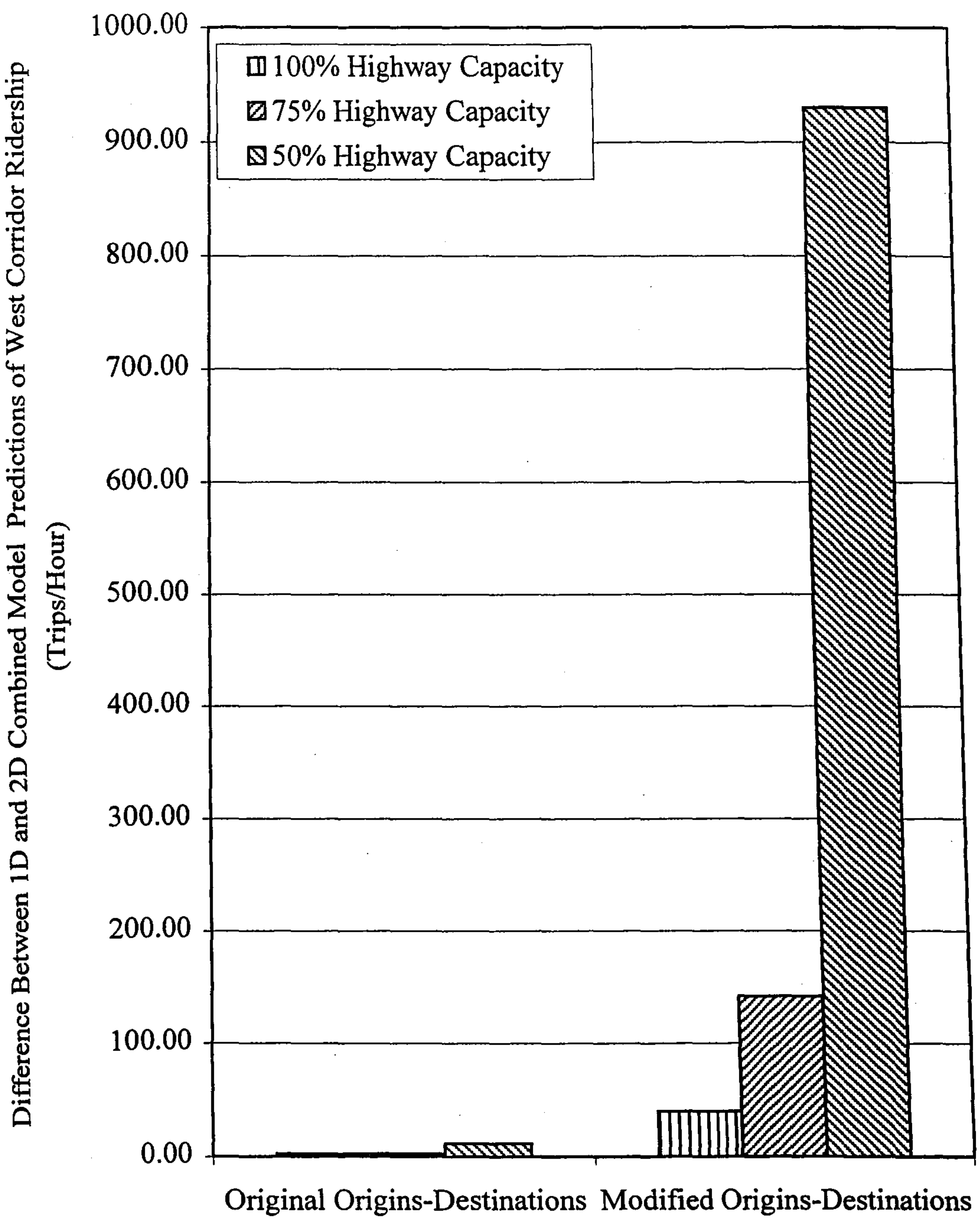


Figure 37: Difference in to-CBD transit trips change by corridors due to transit line extension as predicted by the 1D and 2D Combined Model w.r.t. highway capacity

(Transit line extended in the West corridor)

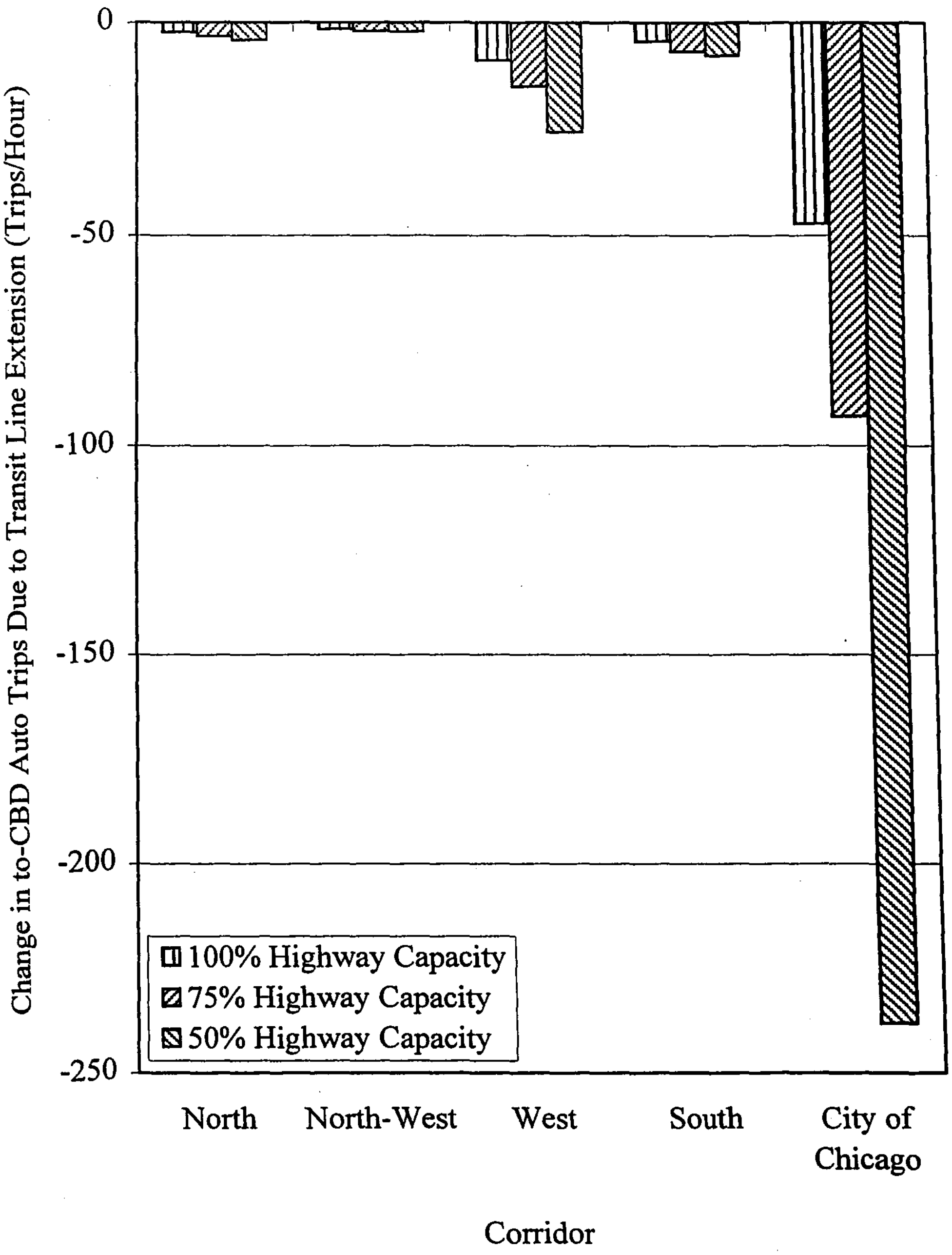


Figure 38: To-CBD auto trips change by corridors due to transit line extension as predicted by the 2D Combined Model w.r.t. highway capacity using original Origins-Destinations

(Transit line extended in the West corridor)

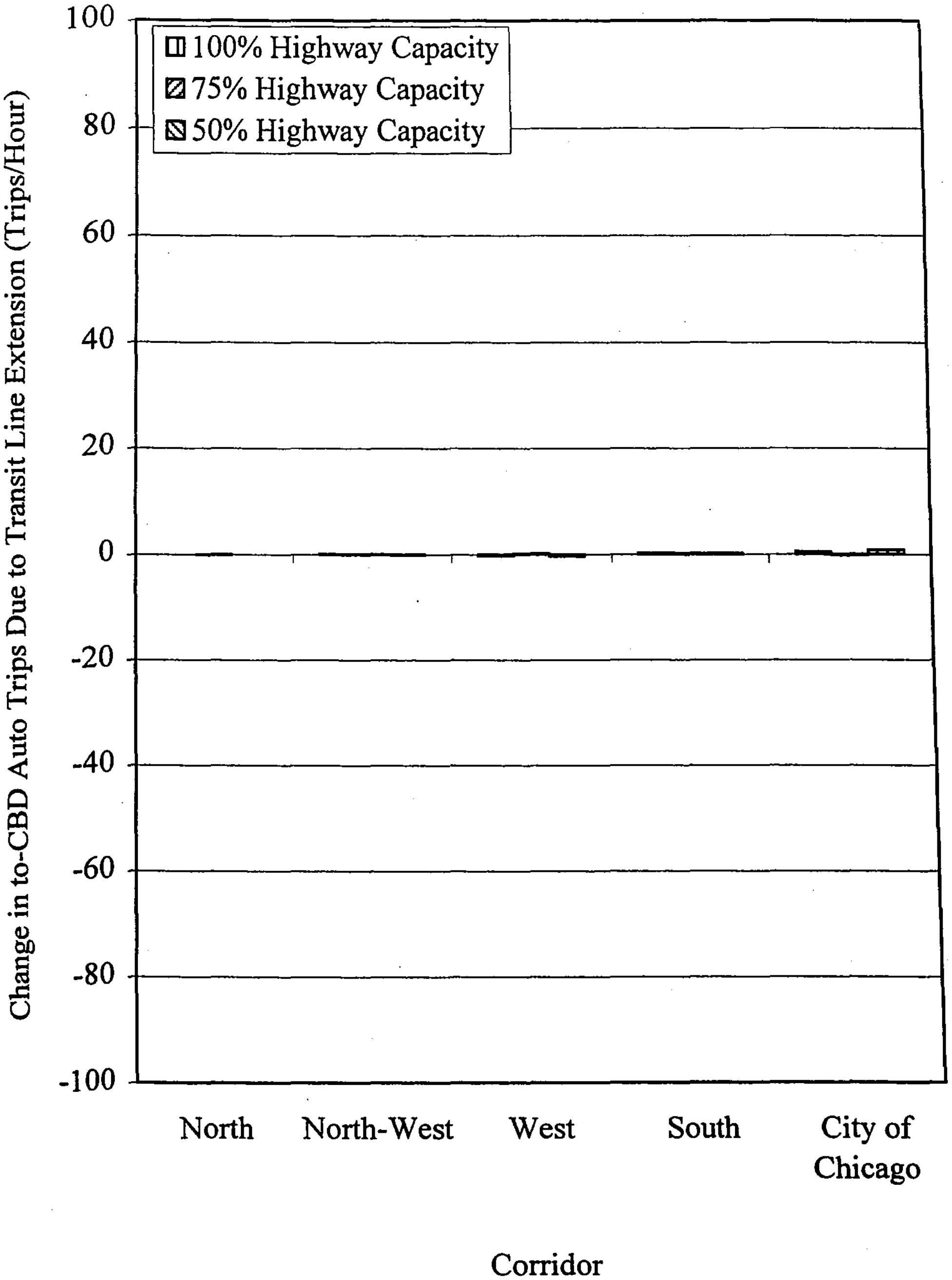


Figure 39: To-CBD auto trips change by corridors due to transit line extension as predicted by the 1D Combined Model w.r.t. highway capacity using original Origins-Destinations

(Transit line was extended in the West corridor)

5. CONCLUSIONS

5.1 Use of Combined Model in the Analysis of Transit Service Enhancements

In order to accommodate regional growth in terms of population, households, employment, major cities in the United States are expanding outward. The expansions are usually radial, towards the periphery of the metropolitan areas. As the residential and employment centers develop and grow along the peripheries of metropolitan areas, travel demand dramatically increases there. To many travelers going to work during the peak period or going to destinations with scarce and costly parking services, or simply when auto network becomes very congested, transit is an alternative option. Naturally peripheral expansion of metropolitan areas means growing demand for transit in the periphery. Outward extension of transit services in order to meet this travel demand is not uncommon in major urban areas.

The use of travel forecasting models is inevitable in feasibility studies, the scenario analyses in order to select the best plan for transit line extensions. A Combined Travel Choice Model, because of its sound mathematical foundation and consistency in the convergence of solutions and superior performance, is a very reliable tool for use in transit extension analysis. However, transportation planners should be careful about the application of Combined Models, as well as traditional travel forecasting procedures, in such analyses.

Depending on the formulation of the Combined Model, i.e. whether or not to consider the destination constraints in the formulation to be rigid, a Combined Model can show quite different behavior in response to transit service extensions. As demonstrated in this research, the Combined Model generally predicts more transit ridership in the areas benefiting from transit line extensions. However, when both the origin constraints and destination constraints are considered in the formulation, a Combined Model will predict decreases in transit ridership in areas not affected by transit line extension at all. Traditional forecasting procedures produce similarly anomalous results.

Moreover, such a formulation of the Combined Model results in a prediction of considerable restructuring of the travel pattern all over the network by all the modes, even though transit line is extended in only one sector of the metropolitan area. Depending on the level of congestion on the accompanying auto network and the number of trips originating from the area benefiting from transit line extension, these effects can be substantial in magnitude. Obviously such a prediction from the model used in the analysis of a transit line extension scenario can be quite unexpected and counter intuitive if attention is not paid to the formulation and the behavior of the model.

On the other hand, if only the origin constraints are considered in the Combined Model formulation, the effect of a transit line extension is confined in the part of the region where the extension takes place. In this case, transit ridership increases and origin-destination and mode choice reorganization are predicted by the model in the area under the influence of the line extension and the travel pattern, as predicted by the model, remain unchanged in the rest of the region. This kind of prediction is more in line with the expectations of the planners.

This study also found that apart from the differences in the prediction of changes in the travel pattern due to a transit line extension, differences in model formulation result in significant differences in the flow predicted by the models. This research reveals that the transportation planners should be careful in the interpretation of model outputs during transit service extension analysis. More importantly, in order to avoid confusion in decision making and misleading conclusions, more attention should be paid to the model formulation. This is especially necessary when the auto network is congested and high volumes of traffic are emerging from zones in the vicinity of transit line extensions.

5.2 Suggestions for Future Research

The current study investigated certain behavioral aspects of the Combined Travel Choice Model with regard to the issue of transit service extension. However, more research can be conducted in related areas. A few suggestions are as follows:

1. The Effect of Auto Access Time: A significant portion of the out-of-vehicle travel time associated with transit trips is the access time to transit stations. Since the major access mode is auto, a majority of the transit passengers interact with the auto

travelers on the same auto network. Effect and extent of such interaction on the regional travel pattern can be investigated using the Combined Model.

2. Effects of Transit Service Improvements: Improvement of service makes transit more attractive to travelers. Transit service improvements may include reduction of fare, higher speeds thereby reducing in-vehicle travel time, more frequent service, which will reduce waiting and transfer time etc. Research can be conducted to explore the response of the Combined Model to such improvements.

3. Use of Explicit Transit Assignment in the Combined Model: The implementation of the Combined Model used in this research did not include any transit assignments. However, explicit transit assignment can help to capture the dynamic restructuring of the market area of each transit line resulting from the extension. A reorganization of market areas of transit line due to line extension may reveal interesting characteristics of the model prediction. Such a study can be undertaken.

4. Effect of Cost Parameters: Cost parameters are used to compute the generalized travel cost and the model makes origin-destination and mode choices based on that generalized cost. The cost parameters are exogenously supplied to the Combined Model solution. A direct effect of a transit line extension is the reduction of the generalized cost of transit travel. Circularly, the optimal cost parameters depend on the transit cost components along with other data. Obviously, an interesting

interaction exists between the selection of optimal cost parameters and transit line extension. Research can be conducted to examine this interaction.

5. Use of Multi-Class Model: In the current study, trips made for all the purposes were considered together. However, it is apparent that trips made for different purposes have different characteristics. A study can be conducted to investigate how trips of different characteristics respond to transit line extensions using a Multi-class Combined Model.

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APPENDICES

APPENDIX A

A.1 1990 Household Travel Survey

A Household Travel Survey was conducted in the Chicago metropolitan area in 1990 by the Chicago Area Transportation Study (CATS), the Metropolitan Planning Organization of the region. The purpose of the survey was to collect information regarding the travel pattern in the region. Households across the Chicago region were randomly selected and requested to participate in the survey. The objective was to identify all the trips made by all adult members of the selected household and to obtain relevant household and personal information.

Households selected to participate in the survey were sent three mailings. The first mailing contained two letters explaining the importance of cooperation, introducing CATS and describing the mechanics of the survey. Two weeks after the introduction letters, the survey packet was sent. A day was designated as the travel day and reports on the trips made on that day were requested. Five days past the designated travel day, a reminder letter was sent to non-responding households.

The survey package contained a letter, instructions and the survey forms. Forms (Figures 40 and 41) were designed to solicit household and personal data that are necessary for the

APPENDIX A (continued)

CITY OF CHICAGO TRAVEL SURVEY 1991

Conducted for: Illinois Department of Transportation
City of Chicago

Conducted by: Chicago Area Transportation Study
Survey Office: (312)793-3467
Toll Free Number: 1-800-637-9125

PLEASE READ THIS FIRST

- Please fill out **PART 1** of the **HOUSEHOLD FORM** first.
- Then fill out **PART 2** of the **HOUSEHOLD FORM** for every household member aged **14 YEARS OR OLDER**.
- Then for every household member aged **14 YEARS OR OLDER** fill out the **TRIP FORMS** for this coming **THURSDAY**.

CONFIDENTIAL

The information obtained in this survey will be accorded confidential treatment, and will be used for statistical purposes only.

HOUSEHOLD FORM-PART 1

A household consists of ALL persons who live together and share the same mailing address.

SAMPLE

How many persons (including yourself) live in this household? 5 Person(s)

Of these people, how many are less than 14 years old? ____ Person(s)

How many are 14 years or older? 4 Person(s)

HOW MANY of the following vehicles are owned or kept at home for use by members of this household?

(Please include all vehicles usually kept at your home overnight.)

2 AUTO(S) ____ MOTORCYCLE(S)

1 VAN(S) & PICKUP(S) (1 Ton or Less) 4 BICYCLE(S)

____ OTHER (specify) ____________

We may need to call you to make sure that we understand all of your answers. What is your telephone number and the first name of the person we should contact? Also, when would be a convenient time to call?

Phone Number: 793 -0000 First Name: David Time: 5:15 AM ☐ PM ☒

Now please answer the questions on **PART 2** of the **HOUSEHOLD FORM** for all persons aged **14 YEARS OR OLDER**.

Figure 40: A Sample Household Form Used in the CATS 1990 Household Travel Survey

(Source: Chicago Area Transportation Study, Chicago, Illinois)

APPENDIX A (continued)

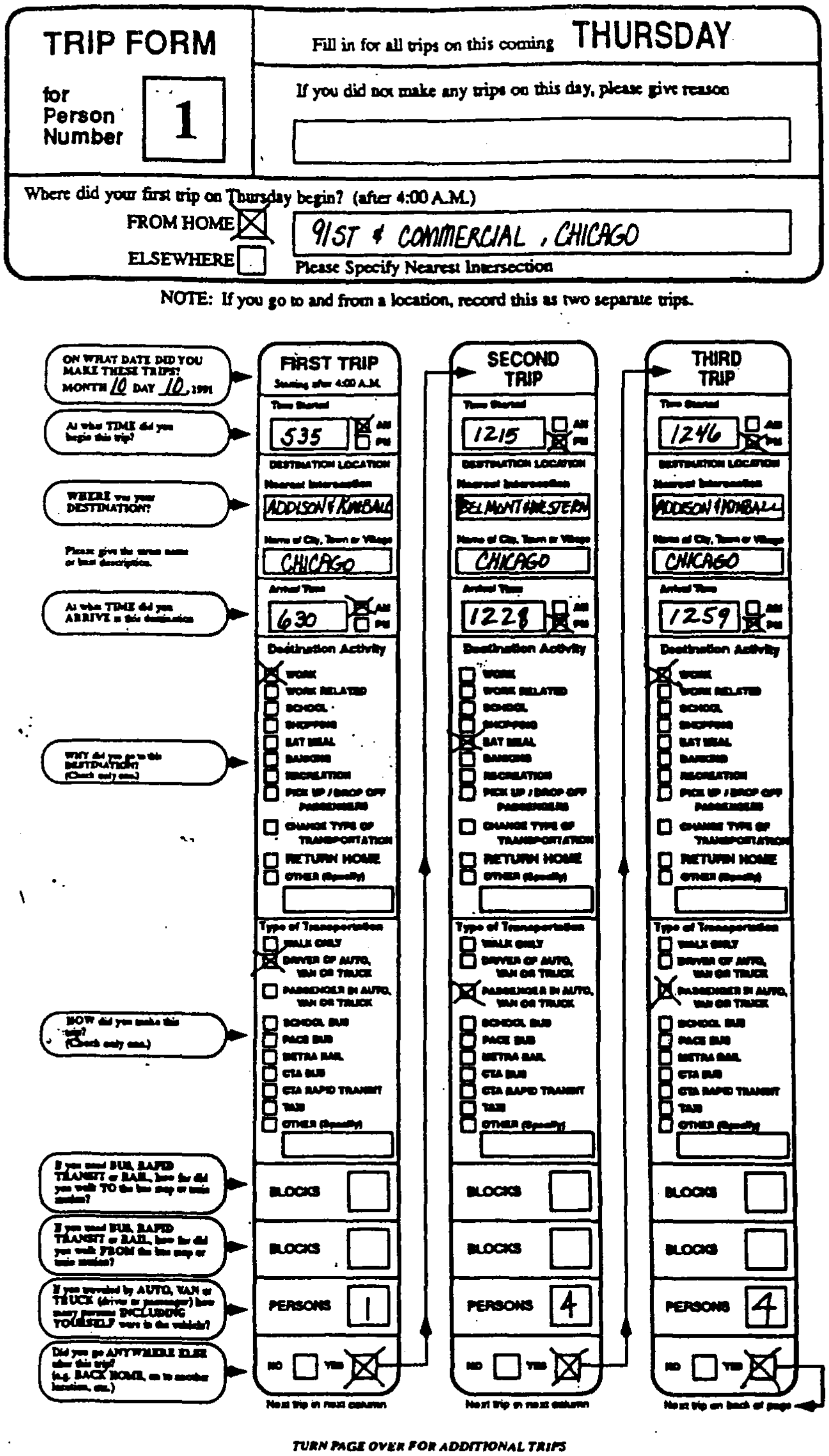

Figure 41: A Sample Trip Form Used in the CATS 1990 Household Travel Survey

(Source: Chicago Area Transportation Study, Chicago, Illinois)

APPENDIX A (continued)

analysis of travel behavior and detail information on each of the trips the respondents made on the travel day, designated as Thursday. Any household member who was 14 years of age or older was requested to respond individually by giving information on trips he/she made between 4:00 AM on the designated travel day through 3:59 AM the next day on separate forms. Each form could hold information for upto seven trips and there were enough forms for four members per household. However, three supplemental forms were included to meet additional need.

A trip was defined as a one-way movement of a person from one location to another. For example, if someone traveled to a place and then returned that would be treated as two separate trips. As another example, if a person drove to a train station and took a train to another station and walked to work that would be counted as three trips.

Responses were obtained from 19,314 households. The information gathered was processed and compiled into three data files: a Household file, a Person file and a Trip file. The set of households returning the completed survey was a sample of the entire population in the Chicago metropolitan area. Adjustment factors were developed to convert the survey trips to reflect the trips made in the region. These factors were developed on a household basis and were termed trip weights. Each trip in the survey was

APPENDIX A (continued)

assigned a household weight that indicates how many trips in the entire population it is equivalent to.

Household file contains household level data such as household location, household population, automobile ownership, household income level etc. Person file has information on the individuals, who are 14 years of age or older, making the trips regardless of whether they made any trip on the survey travel date. The person data included household the person belongs to, relationship of the person with the oldest person in the household, age, gender, employment status, occupational classification, number of trips made etc. Trip file records data about individual trips such as start time, end time, start location, end location, mode of transportation, activity at trip ends, occupancy for trips made by auto, blocks walked to board bus or train for transit trips, trip length, whether a trip was follows by another trip etc. The households, people in the households and the trips made by the people, were related by unique identification numbers.

APPENDIX B

B.1 Preparation of Transit Data

Preparation of transit data is an elaborate process by itself. The general methodology involved can be outlined as follows.

1. Collect data on station, fare structure, station-to-station travel time, frequency of service for all the transit services available.
2. Compute zone-to-zone IVT, fare, waiting time and transfer time, if any, for all the transit services.
3. Calculate zone-to-transit station access time. Access time can be by auto to park-and-ride stations or by walking. If both auto and walk access are available the lesser time-consuming access mode is chosen.
4. Re-compute zone-to-zone transit IVT, fare and the portion of the OVT consisting of waiting time, transfer time and walk egress time after considering the lowest-cost combinations of different transit services, including the possibility of changing transit line during the course of travel. IVT, Fare and the first station-to-destination zone part of OVT will be the ones accumulated along the way.

APPENDIX B (continued)

5. Finally re-calculate zone-to-zone IVT, OVT and fare once gain after taking into account the access time to the first transit station. OVT will be the sum of the access time from the origin zone to the first station and the OVT from that station to the destination zone. IVT and fare will be that between the first transit station and the destination zone. The first transit station should be so selected that the overall generalized cost of travel between the pair of zone under consideration is the minimum achievable.

APPENDIX C

C.1 3D Combined Model

The 3D Combined Model i.e. the combined model with origin and destination constraints and transit share constraint can described as follows.

$$\min g(\mathbf{v},\ \mathbf{P}) = \frac{H}{N}\gamma_1 \sum_a \int_0^{v_a} c_a(x)dx + \frac{1}{N}\gamma_2 \sum_a \int_0^{v_a} k_a(x)dx + \gamma_3 \sum_i \sum_j P_{ijh} w_{ijh} +$$
$$\gamma_4 \sum_i \sum_j P_{ijt} c_{ijt} + \gamma_5 \sum_i \sum_j P_{ijt} k_{ijt} + \gamma_6 \sum_i \sum_j P_{ijt} w_{ijt} + \gamma_7 \sum_i \sum_j P_{ijt} +$$
$$\frac{1}{\mu} \sum_i \sum_j \sum_m P_{ijm} \ln \frac{P_{ijm}}{\overline{P}_i \overline{P}_j} \qquad \text{(C.1)}$$

Subject to

$$\sum_{r \in R_{ij}} f_r = \frac{N}{H} P_{ijh} + T_{ij} : \qquad \forall i, \forall j \qquad \text{(C.2)}$$

$$\sum_j \sum_m P_{ijm} = \overline{P_i} : \qquad \forall i \qquad \text{(C.3)}$$

$$\sum_i \sum_m P_{ijm} = \overline{P_j} : \qquad \forall j \qquad \text{(C.4)}$$

$$\sum_i \sum_j P_{ijt} = \overline{P}_t \qquad \text{(C.5)}$$

$$f_r \geq 0 : \qquad \forall r \qquad \text{(C.6)}$$

By definition

$$v_a = \sum_i \sum_j \sum_{r \in R_{ij}} f_r \delta_r^a : \qquad \forall a \qquad \text{(C.7)}$$

APPENDIX C (continued)

The Lagrangian is as follows:

$$L = \frac{H}{N}\gamma_1 \sum_a \int_0^{v_a} c_a(x)dx + \frac{1}{N}\gamma_2 \sum_a \int_0^{v_a} k_a(x)dx + \gamma_3 \sum_i \sum_j P_{ijh} w_{ijh} + \gamma_4 \sum_i \sum_j P_{ijt} c_{ijt} + \gamma_5 \sum_i \sum_j P_{ijt} k_{ijt} +$$
$$\gamma_6 \sum_i \sum_j P_{ijt} w_{ijt} + \gamma_7 \sum_i \sum_j P_{ijt} + \frac{1}{\mu} \sum_i \sum_j \sum_m P_{ijm} \ln \frac{P_{ijm}}{\overline{P_i}\,\overline{P_j}} + \sum_i \sum_j u_{ijh} \left(\frac{N}{H} P_{ijh} + T_{ij} - \sum_{r \in R_{ij}} f_r \right) +$$
$$\sum_i \alpha_i \left(\overline{P_i} - \sum_j \sum_m P_{ijm} \right) + \sum_j \beta_j \left(\overline{P_j} - \sum_i \sum_m P_{ijm} \right) + \eta_t \left(\overline{P_t} - \sum_i \sum_j P_{ijt} \right) + \sum_i \sum_j \sum_{r \in R_{ij}} \theta_r (-f_r) \quad \text{(C.8)}$$

where η_t is a new Lagrange multiplier related to constraint (C.5).

Partial derivative of L w.r.t. f_r gives the following equation.

$$\frac{\partial L}{\partial f_r} = \frac{H}{N}\gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{N}\gamma_2 \sum_a k_a(v_a)\delta_r^a - u_{ijh} - \theta_r = 0 : \qquad r \in R_{ij} \qquad \text{(C.9)}$$

If $f_r > 0$, by complementary slackness $\theta_r = 0$. Thus Equation (C.9) yields :

$$\gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{H}\gamma_2 \sum_a k_a(v_a)\delta_r^a = \frac{N}{H} u_{ijh} = \tilde{u}_{ijh} \qquad \text{(C.10)}$$

If $f_r = 0$, by complementary slackness $\theta_r \geq 0$. Then Equation (2.9) reduces to:

$$\gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{H}\gamma_2 \sum_a k_a(v_a)\delta_r^a = \frac{N}{H} u_{ijh} + \frac{N}{H}\theta_r = \hat{u}_{ijh} \qquad \text{(C.11)}$$

Now, the partial derivative of L w.r.t. P_{ijh} gives the following.

APPENDIX C (continued)

$$\frac{\partial L}{\partial P_{ijh}} = \gamma_3 w_{ijh} + \frac{1}{\mu}\left(\ln\frac{P_{ijh}}{\overline{P}_i \overline{P}_j} + 1\right) + \frac{N}{H} u_{ijh} - \alpha_i - \beta_j = 0$$

$$\Rightarrow \qquad \gamma_3 w_{ijh} + \frac{1}{\mu}\left(\ln\frac{P_{ijh}}{\overline{P}_i \overline{P}_j} + 1\right) + \gamma_1 \sum_a c_a(v_a)\delta_r^a + \frac{1}{H}\gamma_2 \sum_a k_a(v_a)\delta_r^a - \alpha_i - \beta_j = 0$$

$$\Rightarrow \qquad P_{ijh} = \overline{P}_i \overline{P}_j \exp(-\mu C_{ijh}) \exp(\mu\alpha_i - 1) \exp(\mu\beta_j)$$

$$\Rightarrow \qquad P_{ijh} = a_i \overline{P}_i b_j \overline{P}_j \exp(-\mu C_{ijh}) \qquad \text{(C.12)}$$

Again, the partial derivative of L w.r.t. P_{ijt} yields:

$$\frac{\partial L}{\partial P_{ijt}} = \gamma_4 c_{ijt} + \gamma_5 k_{ijt} + \gamma_6 w_{ijt} + \gamma_7 + \frac{1}{\mu}\left(\ln\frac{P_{ijh}}{\overline{P}_i \overline{P}_j} + 1\right) - \alpha_i - \beta_j - \eta_t = 0$$

$$\Rightarrow \quad P_{ijt} = \overline{P}_i \overline{P}_j \exp(-\mu C_{ijt}) \exp(\mu\alpha_i - 1) \exp(\mu\beta_j) \exp(\mu\eta_t)$$

$$\Rightarrow \quad P_{ijt} = a_i \overline{P}_i b_j \overline{P}_j c_t \exp(-\mu C_{ijh}) \qquad \text{(C.13)}$$

where

$$c_t = \exp(\mu\eta_t) \qquad \text{(C.14)}$$

= a measure of attractiveness for transit

Equation (C.3) can be written as follows.

APPENDIX C (continued)

$$\sum_j P_{ijh} + \sum_j P_{ijt} = \overline{P}_i$$

$$\Rightarrow \quad a_i \overline{P}_i \left(\sum_j b_j \overline{P}_j \exp\left(-\mu C_{ijh}\right) \right) + a_i \overline{P}_i c_t \left(\sum_j b_j \overline{P}_j \exp\left(-\mu C_{ijt}\right) \right) = \overline{P}_i$$

$$\Rightarrow \quad a_i = \frac{1}{\sum_j b_j \overline{P}_j \exp\left(-\mu C_{ijh}\right) + c_t \sum_j b_j \overline{P}_j \exp\left(-\mu C_{ijt}\right)} \qquad \text{(C.15)}$$

Equation (C.4) can be rearranged as follows.

$$\sum_i P_{ijh} + \sum_i P_{ijt} = \overline{P}_j$$

$$\Rightarrow \quad b_j \overline{P}_j \left(\sum_i a_i \overline{P}_i \exp\left(-\mu C_{ijh}\right) \right) + b_j \overline{P}_j c_t \left(\sum_i a_i \overline{P}_i \exp\left(-\mu C_{ijt}\right) \right) = \overline{P}_j$$

$$\Rightarrow \quad b_j = \frac{1}{\sum_i a_i \overline{P}_i \exp\left(-\mu C_{ijh}\right) + c_t \sum_i a_i \overline{P}_i \exp\left(-\mu C_{ijt}\right)} \qquad \text{(C.16)}$$

Equation (C.5) gives the following.

$$\Rightarrow \quad \sum_i \sum_j P_{ijt} = \overline{P}_t$$

$$\Rightarrow \quad \sum_i \sum_j a_i \overline{P}_i b_j \overline{P}_j c_t \exp\left(-\mu C_{ijt}\right) = \overline{P}_t$$

$$\Rightarrow \quad c_t = \frac{\overline{P}_t}{\sum_i \sum_j a_i \overline{P}_i b_j \overline{P}_j \exp\left(-\mu C_{ijt}\right)} \qquad \text{(C.17)}$$

APPENDIX D

D.1 Definitions

1D Combined Model

In this thesis 1D Combined Model is the combined model with origin constraints only.

1D Existing Transit Solution

Solution of the 1D Combined Model with the Existing Transit Data.

1D Extended Transit Solution

Solution of the 1D Combined Model with the Extended Transit Data.

2D Combined Model

Combined model with both origin constraints and destination constraints.

2D Existing Transit Solution

Solution of the 2D Combined Model with the Existing Transit Data.

2D Extended Transit Solution

Solution of the 2D Combined Model with the Extended Transit Data.

APPENDIX D (continued)

3D Combined Model

Combined model with origin constraints, destination constraints and transit share constraint.

Access Time

Time needed for a traveler to go to a transit station.

Affected Zones

The set of zones affected by transit line extension in terms of cost of transit travel.

Analysis Period

The period of time for a transportation network is analyzed suing a model.

Best Lower Bound

The maximum of the optimal value of the sub-problem objective function in Evans algorithm.

Boarding Zone

A network zone where a traveler goes in order to board a transit service.

APPENDIX D (continued)

CATS Transit Data

Transit data used in this study that was prepared by CATS.

CBD Transit Share

Fraction of CBD the bound trips made by transit in a certain period of time. In this thesis the time period is the morning peak period.

Chicago Region

Chicago metropolitan area.

Combined Model

A network equilibrium model used for travel forecasting that combines the origin-destination choice, mode choice and route choice phases of the traditional separate and sequential models together.

Contribution Factor

What fraction of a regional network zone contributes to a sketch network zone.

APPENDIX D (continued)

Cost Parameters

Weights used to convert IVT, OVT and monetary cost of a trip into generalized travel cost.

Destinations (Ds)

Total number of trip coming into different network zones.

Destination Attractiveness Factors

A set of factors giving a measure of the attractiveness of network zones as trip destination.

Destination Constraints

A set of constraints used in combined model formulation that ensure that the trip matrices obtained from the model solution will be such that the total number of trips entering each zone will be equal to an observed value for that zone.

Existing Transit Data

Transit data for the existing transit network.

APPENDIX D (continued)

Extended Transit Data

Transit data for the extended transit network.

Extension Corridor

The transit corridor, where some transit lines are extended.

Generalized Cost

Weighted combination of IVT, OVT and monetary cost associated with a trip.

Main Problem

The optimization problem that constitutes the combined model.

Modified Origins-Destinations

A Set of origins (Os) and destinations (Ds) used in this study and prepared by increasing the origins (Os) in the network zones affected by transit line extension and adjusting the destinations (Ds) accordingly.

Morning Peak Period

In this study morning peak period was from 6:45 Am to 8:45 AM.

APPENDIX D (continued)

Origins (Os)

Total number of trips going out of different network zones.

Origin Attractiveness Factors

A set of factors indicating the attractiveness of network zones as trip origin.

Origin Constraints

A set of constraints used in the formulation of the combined model so that the trip matrices produced by solving the model will be such that for each network zone the total number of trips leaving that zone will equal a pre-estimated value.

Original Origins-Destinations

A Set of origins (Os) and destinations (Ds) used in this study and prepared by using data obtained from CATS.

Partially Linearized Objective Function

Linear approximation of the non-linear convex objective function of the combined model w.r.t. link flows.

APPENDIX D (continued)

Regional Network

A transportation network encompassing the Chicago metropolitan area that has 1790 zones, 12982 nodes and 39018 links.

Regional Transit Data

Transit data for the regional network with the existing transit network.

Sketch Network

A transportation network, representing the Chicago metropolitan area, consisting of 387 zones, 933 nodes and 2950 links.

Sub-problem

An optimization problem where the partially linearized objective function of the main problem is minimized subject to the main problem constraints.

TransLab Transit Data

Transit data used in this research prepared by the TransLab.

Transit Bias

A parameter, treated as a component of cost of transit travel, used to incorporate the relative attractiveness of transit over other modes of travel.

APPENDIX D (continued)

Transit Corridor

A collection of network zones that contribute traffic to a group of geographically close transit service lines.

Transit Cost Components

Transit IVT, OVT and fare.

Transit Data

Zone to zone transit IVT, OVT and fare for all the zone pairs in a transportation network.

Transit Line

A bus route or a train line. In this thesis, it is a Metra Commuter Rail line.

Transit Share

Fraction of trips made in a transportation network using transit in certain period of time. In this document the period of time is the morning peak period.

Transit Share Constraint

Constraint used in a form of combined model ensuring that the model will predict a transit share equal to some observed value.

APPENDIX D (continued)

Trip Table

Number of trips made by using a mode of travel between each pair of zones of a transportation network.